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# Theoretical design approach of four-dimensional piecewise-linear multi-wing hyperchaotic differential dynamic system

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ABSTRACT

This paper reports a novel approach for constructing four-dimensional piecewise-linear multi-wing hyperchaotic differential dynamic system. First, two basic four-dimensional linear systems are obtained. Then, by switching control based on Shilnikov theorem, the double-wing hyperchaotic attractors can be generated. Thirdly, by $m$th shifting transformation of four-dimensional linear systems, multi-wing hyperchaotic attractors are realized. The designed systems are chaotic in the sense of Smale horseshoe. To confirm the existence of hyperchaotic system, Lyapunov exponent spectrums are further investigated.



## 1. Introduction

Recently, a large number of chaotic systems have been investigated and studied successively because of the great applications in secure communication, data encryption, flow dynamics as well as in engineering applications, including Lorenz system, Chua system, generalized chaotic Lorenz system family [1–4]. In addition, the famous Chua system can generate various multi-scroll chaotic attractors based on the extension of saddle-focus equilibrium points with index 2 [5,6]. Similarly, many other systems can also create various complex multi-scroll attractors [7–10]. Recently, research for more complex chaotic systems has led to the finding of multi-wing chaotic attractors [11–13]. Compared with the double-wing and multi-wing chaotic systems, complex grid multi-wing hyperchaotic systems exhibit more complicated dynamical behaviors and better performance in many other technological application fields, such as secure communication systems, where the information is encrypt and the security performance can be improved by using these complex grid multi-wing hyperchaotic signals. One may ask whether or not there is a general approach for constructing various complex grid multi-wing hyperchaotic systems? This paper gives a positive answer to the question.

It is well known that Shilnikov theorem can be adopted as the approach for judging whether chaos exists or not in a certain autonomous system. For instance, the three-dimensional piecewise-linear Lorenz system and Chua system are analyzed based on Shilnikov theorem, respectively, and the existence of Smale horseshoe is also rigorously proved [14,15]. On the other hand, the Shilnikov theorem could also play a role in theoretical basis and realization means for constructing chaotic and hyperchaotic systems [16–19].

In this paper, from two basic four-dimensional linear systems, based on Shilnikov theorem, one proposes a new general approach to construct variable number of multi-wing hyperchaotic attractors.

The rest of this paper is organized as follows. In Section 2, two basic four-dimensional linear systems are obtained. In Section 3, the super-heteroclinic loop is found via switching control and two-piecewise-linear hyperchaotic system is constructed. In Section 4, by $m$th shift transformation of linear systems, multi-piecewise-linear hyperchaotic systems are further obtained for generating multi-wing hyperchaotic attractors. In Section 5, Lyapunov exponent spectrums are further investigated. Conclusions are finally drawn in Section 6.

## 2. Two basic four-dimensional linear systems

Consider the following two four-dimensional fundamental linear systems:

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$$\begin{pmatrix}\dot{x}_1\\ \dot{y}_1\\ \dot{z}_1\\ \dot{u}_1\end{pmatrix}\begin{pmatrix}-a & ac & \sqrt{abc} & 0\\ -1 & c & 0 & 1\\ 0 & 2\sqrt{abc} & -b & 0\\ e & p & 0 & q\end{pmatrix}\begin{pmatrix}x_1\\ y_1\\ z_1\\ u_1\end{pmatrix} = J_1\begin{pmatrix}x_1\\ y_1\\ z_1\\ u_1\end{pmatrix} \quad (1)$$

$$\begin{pmatrix}\dot{x}_1\\ \dot{y}_1\\ \dot{z}_1\\ \dot{u}_1\end{pmatrix} = \begin{pmatrix}-a & ac & -\sqrt{abc} & 0\\ -1 & c & 0 & 1\\ 0 & -2\sqrt{abc} & -b & 0\\ e & p & 0 & q\end{pmatrix}\begin{pmatrix}x_1\\ y_1\\ z_1\\ u_1\end{pmatrix} = J_1\begin{pmatrix}x_1\\ y_1\\ z_1\\ u_1\end{pmatrix} \quad (2)$$

where $a = 20$, $b = 5$, $c = 10$, $e = 0.6$,, $p = 0.4$, $q = 1.2$. Notice that the equilibrium points of systems (1) and (2) are $O_1 = (0, 0, 0, 0)$ and $O_2 = (0, 0, 0, 0)$, and the corresponding eigenvalues at $O_1$ and $O_2$ are same as $\gamma_1 = -18.2194$, $\gamma_2 = 2.1701$, and $\sigma \pm j\omega = 1.1246 \pm j10.2943$, respectively. Obviously, $O_1$ and $O_2$ are saddle-focus equilibrium points with index-2. The eigenvectors corresponding to eigenvalues $\gamma_1 = -18.2194$, $\gamma_2 = 2.1701$, and $\sigma \pm j\omega = 1.1246 \pm j10.2943$ at the equilibrium point $O_1 = (0, 0, 0, 0)$ are solved by:

$$\begin{cases}\mu_1 = \xi^{(1)}_{\gamma_1} = \begin{pmatrix}\xi^{(1)}_{1\gamma_1}\\ \xi^{(1)}_{2\gamma_1}\\ \xi^{(1)}_{3\gamma_1}\\ \xi^{(1)}_{4\gamma_1}\end{pmatrix} = \begin{pmatrix}0.9839\\ 0.0360\\ -0.1721\\ -0.0311\end{pmatrix},\\ \nu_1 = \xi^{(1)}_{\gamma_2} = \begin{pmatrix}\xi^{(1)}_{1\gamma_2}\\ \xi^{(1)}_{2\gamma_2}\\ \xi^{(1)}_{3\gamma_2}\\ \xi^{(1)}_{4\gamma_2}\end{pmatrix} = \begin{pmatrix}0.7967\\ 0.0369\\ 0.3253\\ 0.5080\end{pmatrix}\\ \eta_1 = \xi^{(1)}_R \pm j\xi^{(1)}_I = \begin{pmatrix}\xi^{(1)}_{1R}\\ \xi^{(1)}_{2R}\\ \xi^{(1)}_{3R}\\ \xi^{(1)}_{4R}\end{pmatrix} \pm j\begin{pmatrix}\xi^{(1)}_{1I}\\ \xi^{(1)}_{2I}\\ \xi^{(1)}_{3I}\\ \xi^{(1)}_{4I}\end{pmatrix}\\ \quad = \begin{pmatrix}0.9285\\ 0.0413\\ 0.3583\\ 0.0017\end{pmatrix} \pm j\begin{pmatrix}0\\ 0.0543\\ -0.0413\\ -0.0557\end{pmatrix}\end{cases} \quad (3)$$

Similarly, the eigenvectors corresponding to the eigenvalues $\gamma_1 = -18.2194$, $\gamma_2 = 2.1701$, and $\sigma \pm j\omega = 1.1246 \pm j10.2943$ at the equilibrium point $O_2 = (0, 0, 0, 0)$ are obtained as follows:

$$\begin{cases}\mu_2 = \xi^{(2)}_{\gamma_1} = \begin{pmatrix}\xi^{(2)}_{1\gamma_1}\\ \xi^{(2)}_{2\gamma_1}\\ \xi^{(2)}_{3\gamma_1}\\ \xi^{(2)}_{4\gamma_1}\end{pmatrix} = \begin{pmatrix}0.9839\\ 0.0360\\ 0.1721\\ -0.0311\end{pmatrix},\\ \nu_2 = \xi^{(2)}_{\gamma_2} = \begin{pmatrix}\xi^{(2)}_{1\gamma_2}\\ \xi^{(2)}_{2\gamma_2}\\ \xi^{(2)}_{3\gamma_2}\\ \xi^{(2)}_{4\gamma_2}\end{pmatrix} = \begin{pmatrix}0.7967\\ 0.0369\\ -0.3253\\ 0.5080\end{pmatrix}\\ \eta_2 = \xi^{(2)}_R \pm j\xi^{(2)}_I = \begin{pmatrix}\xi^{(2)}_{1R}\\ \xi^{(2)}_{2R}\\ \xi^{(2)}_{3R}\\ \xi^{(2)}_{4R}\end{pmatrix} \pm j\begin{pmatrix}\xi^{(2)}_{1I}\\ \xi^{(2)}_{2I}\\ \xi^{(2)}_{3I}\\ \xi^{(2)}_{4I}\end{pmatrix}\\ \quad = \begin{pmatrix}0.9285\\ 0.0413\\ -0.3583\\ 0.0017\end{pmatrix} \pm j\begin{pmatrix}0\\ 0.0543\\ 0.0413\\ -0.0557\end{pmatrix}\end{cases} \quad (4)$$

It is noticed that the real part of eigenvalues $\gamma_1$, $\gamma_2$ and $\sigma \pm j\omega$ satisfies the Shilnikov inequality, i.e., $|\gamma_1| > |\sigma| > 0, |\gamma_2| > |\sigma| > 0$. Thus, based on Shilnikov theorem, one can build a super-heteroclinic loop in four-dimensional space by choosing the stable manifold corresponding to $\gamma_1$ and the unstable manifold corresponding to $\sigma \pm j\omega$.

According to (3), the stable manifold $E^S(O_1)$ corresponding to eigenvalue $\gamma_1 = -18.2194$ and the unstable manifold $E^U(O_1)$ corresponding to eigenvalue $\sigma \pm j\omega = 1.1246 \pm j10.2943$ at the equilibrium point $O_1 = (0, 0, 0, 0)$ of system (1) are given by:

$$\begin{cases}E_S(O_1): \dfrac{x}{l_1} = \dfrac{y}{m_1} = \dfrac{z}{n_1} = \dfrac{u}{p_1}\\ E^U(O_1): A_1x + B_1y + C_1z + D_1u = 0\end{cases} \quad (5)$$

where $l_1 = 0.9839$, $m_1 = 0.0360$, $n_1 = -0.1721$, $p_1 = -0.0311$, $A_1 = -32.4816$, $B_1 = 18.2194$, $C_1 = 11.4998$ and $D_1 = -1$.

Similarly, according to (4), the stable manifold $E^S(O_2)$corresponding to eigenvalue $\gamma_1 = -18.2194$ and the unstable manifold $E^U(O_2)$ corresponding to eigenvalue $\sigma \pm j\omega = 1.1246 \pm j10.2943$ at the equilibrium point $O_2 = (0, 0, 0, 0)$ of system (2) are solved by:

$$\begin{cases}E^S(O_2): \dfrac{x}{l_2} = \dfrac{y}{m_2} = \dfrac{z}{n_2} = \dfrac{u}{p_2}\\ E^U(O_2): A_2x + B_2y + C_2z + D_2u = 0\end{cases} \quad (6)$$

where $l_2 = 0.9839$, $m_2 = 0.0360$, $n_2 = 0.1721$, $p_2 = -0.0311$, $A_2 = -32.4816$, $B_2 = 18.2194$, $C_2 = -11.4998$ and $D_2 = -1$.

Compared parameters in (5) with (6), One can see the symmetry between (5) and (6), which is the premise of constructing the two-piecewise-linear hyperchaotic system with super-heteroclinic loop and Smale horseshoe.

## 3. Constructing four-dimensional piecewise-linear hyperchaotic system

According to above two basic four-dimensional linear systems (1) and (2), one can further find the super-heteroclinic loop and calculate its necessary parameters via switching control strategy. The equilibrium points of systems (1) and (2) are $O_1 = (0,0,0,0)$ and