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Theoretical design and circuit realization of complex grid multi-wing chaotic system

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ABSTRACT

This paper further investigates a novel method for generating complex grid multi-wing chaotic attractors by double-mirror symmetry conversion both in z direction and in y direction. First, multi-wing chaotic attractors are obtained from a three-dimensional quadratic chaotic system by designing an even-symmetry multi-segment quadratic function to extend the number of saddle-focus equilibrium points with index 2 in y direction. Second, the principle of mirror symmetry conversion for generating grid multi-wing is given. Third, based on the principle, by the double-mirror symmetry conversion with respect to z-axis and y-axis, respectively, intended grid $n \times m$-wing chaotic systems can be obtained. Finally, improved module-based circuits are designed for generating grid multi-wing chaotic attractors, which has demonstrated the feasibility of the proposed approaches.

1. Introduction

Recently, chaos theory and its applications in nonlinear system, information science as well as in engineering applications have been one of the hot research focuses. Since the discovery of the Lorenz system by the eminent scholar, Lorenz during his study in 1963, the famous butterfly attractor has become a paradigm in chaos [1–3]. The generalized Lorenz system family, such as Chen and Lü systems, have been discovered successively which can generate topologically non-equivalent various double-wing chaotic attractors [4–6].

It is well known that the generalized 3-D Lorenz system family posses the characteristics of double-wing butterfly-shaped attractors and the invariance under the transformation $(x, y, z) \rightarrow (-x, -y, z)$ [1,4,7–10]. In order to break the limitation of having only double-wing in an attractor, by introducing duality-symmetric multi-segment quadratic function, multi-wing attractors from Lorenz system family can be obtained [11–13].

It is worth noting that compared with the double-wing and multi-wing chaotic systems, complex grid multi-wing chaotic systems exhibit more complicated dynamical behaviors and better performance in many other technological application fields, such as secure communication systems, where the information is encrypt and the security performance can be improved by using these complex grid multi-wing chaotic signals. One may ask whether or not there is a possible way further to break such a limitation of having only double-wing and multi-wing chaotic attractors and construct various complex grid multi-wing chaotic Lorenz-like systems? This paper gives a positive answer to the question.

This paper further investigates some other new methods for generating complex grid multi-wing chaotic system. Specifically, via switching control, mirror and double-mirror symmetry conversions, two classes of complex grid $n \times m$-wing chaotic systems are respectively obtained, and the improved module-based circuits are also designed for generating various grid multi-wing chaotic attractors. One characteristic of the proposed approaches lies in their generality, which is also suitable for constructing other grid multi-wing chaotic systems. Both numerical simulation and circuit implementation have demonstrated the feasibility and effectiveness of the proposed approaches.

The rest of this paper is organized as follows. In Section 2, n-wing chaotic system equipped with duality-symmetric multi-segment quadratic function is constructed from a modified double-wing chaotic system. In Section3, principle of mirror symmetry conversion for dynamical system is given. In Section 4, construction of grid multi-wing chaotic systems via mirror and double-mirror symmetry conversions is achieved. In Section 5, the bifurcation and Lyapunov exponent (LE) are further calculated. In Section 6, an improved module-based circuit is designed and implemented for generating grid $n \times m$-wing chaotic attractors. Finally, conclusions are drawn in Section 7.

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2. Multi-wing chaotic attractors from a double-wing chaotic system

Recall a three-dimensional quadratic double-wing chaotic system [14]:

$$\begin{cases} \dfrac{dx}{d\tau} = -ax + yz \\ \dfrac{dy}{d\tau} = -x + cy \\ \dfrac{dz}{d\tau} = -bz + dy^2 \end{cases} \tag{1}$$

here $a = 20$, $b = 5$, $c = 10$, $d = 7$. The numerical simulation results of system (1) are depicted in Fig. 1.

In the following circuit design, the power supply for all operational amplifiers is $\pm 15V$, and the saturation value for all operational amplifiers is $E_{sat} = \pm 13.5V$. From Fig. 1, the linear dynamic range of state variables x, y, z are in $[-500, 500]$, $[-30, 30]$ and $[0, 500]$, respectively, which exceeds the linear dynamic range of all operational amplifiers. Therefore, the variable-scale reduction in Eq. (1) is necessary.

Let variable-scale reduction factor be E. After some simplified treatment, Eq. (1) is transferred as follows:

$$\begin{cases} \dfrac{dx}{d\tau} = -ax + \dfrac{1}{E}yz \\ \dfrac{dy}{d\tau} = -x + cy \\ \dfrac{dz}{d\tau} = -bz + \dfrac{d}{E}y^2 \end{cases} \tag{2}$$

here $a = 20$, $b = 5$, $c = 10$, $d = 7$, and variable-scale reduction factor $E = 1/80$. Based on Eq. (2), the numerical simulation results of double-wing chaotic attractors are shown in Fig. 2.

To extend the saddle-focus equilibrium points with index 2 in Eq. (2), a novel parameterized-adjustable duality-symmetric multi-segment quadratic function expressed in Eq. (3) will be constructed, which makes it possible that multi-wing chaotic attractors can be obtained from the modified system.

$$f(y) = H_0 y^2 + \sum_{i=1}^{N} H_i\,[sgn(y + C_i) - sgn(y - C_i) - 2] \tag{3}$$

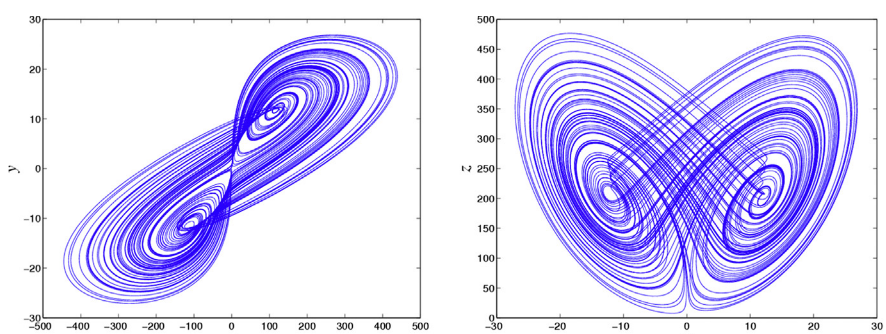
here $N \geq 1$ is a positive integer, H_0, H_i and C_i are recursive parameters, obtained by

$$\begin{cases} H_0 = \dfrac{(1/E)}{k} \\ H_i = \dfrac{AE}{k_i} \\ C_i = 0.5(n+1)\dfrac{AE}{k} \end{cases} \tag{4}$$

here $i = 1, 2, 3, \ldots, N$.

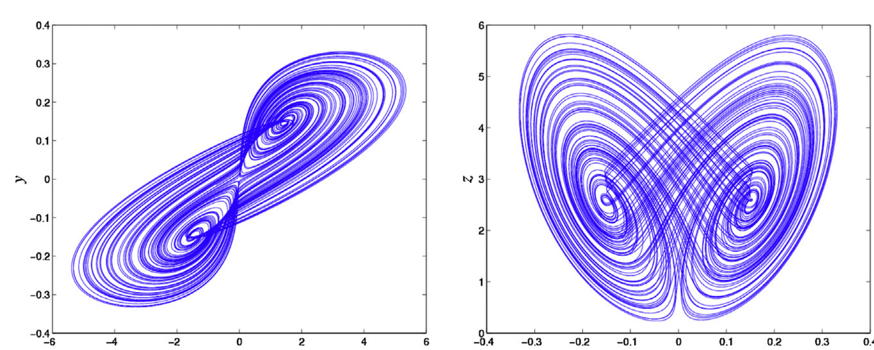
In order to generate multi-wing chaotic attractors, the square term $(d/E)y^2$ in the third state equation of Eq. (2) is replaced by Eq. (3). The modified equations are expressed in

$$\begin{cases} \dfrac{dx}{d\tau} = -ax + \dfrac{1}{E}yz \\ \dfrac{dy}{d\tau} = -x + cy \\ \dfrac{dz}{d\tau} = -bz + f(y) \end{cases} \tag{5}$$



(a) Double-wing attractor in $x - y$ plane (b) Double-wing attractor in $y - z$ plane

Fig. 1. Double-wing chaotic attractors before variable-scale reduction.



(a) Double-wing attractor in $x - y$ plane (b) Double-wing attractor in $y - z$ plane

Fig. 2. Double-wing chaotic attractors after variable-scale reduction.