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# Investigation of relativistic self-focusing of Hermite-cosine-Gaussian laser beam in collisionless plasma

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ABSTRACT

Relativistic self-focusing of Hermite-cosine-Gaussian (HcosG) laser beam in plasma has been investigated. The distribution of HcosG laser field in the plasma is closely related to decentered parameter. The differential equation for the beam width parameter is derived by following Wentzel–Kramers–Brillouin (WKB) and paraxial approximation through parabolic wave equation approach. The behavior of beam width parameter with distance of propagation is investigated at different values of plasma density, decentered parameter and intensity parameter. Our results reveal that the laser beam is more focused at lower values of decentered and intensity parameter. Further, a strong oscillatory behavior of the beam width parameter during laser propagation in collisionless plasma is seen.



## 1. Introduction

The interaction of high power laser beams with plasmas occupies a unique place in the field of research due to wide-ranging applications in laser-driven fusion, laser-driven charged particle accelerators, X-ray lasers etc. [1–3]. For such applications, it is necessary that the laser beam is highly powerful, intense and propagates for extended distances without divergence, resulting in various nonlinear phenomena like self-focusing etc. Therefore, it is important to study such phenomena numerically and analytically. Since the first investigation on self-trapping of optical beams was reported by Askaryan [4] and later the self-focusing of a laser beam in plasma was examined by a number of authors [5–7]. The self-focusing, self-trapping and filamentation of laser has been theoretically investigated by Akhmanov et al. [8] and developed by Sodha et al. [9]. The self-focusing of quadruple Gaussian laser beam in inhomogeneous magnetized plasma with ponderomotive nonlinearity and linear absorption confirms that converging beam shows oscillatory convergence whereas diverging beam shows oscillatory divergence. Further, the beam is more focused at lower intensity under the influence of linear absorption and magnetic field [10]. Kant and Wani [11] reported that plasma density ramp, decentered parameter and linear absorption coefficient are in such a way that they change the nature of self-focusing/defocusing of the laser beam significantly. The absorption weakens the self-focusing effect and density transition sets an early and stronger self-focusing of cosh-Gaussian laser beam. The propagation of circularly polarized quadruple Gaussian laser beam can be studied in three different regimes viz steady divergence, oscillatory divergence and self-focusing regime by taking in to account the effect of magnetic field. The magnetic field improves self-focusing for extraordinary mode but, weakens the effect for ordinary mode [12]. An intense laser beam undergoes self-focusing because of the relativistic mass and ponderomotive effects. The self-focused laser diffracts and focuses more and more during propagation of laser beam. As the plasma density increases, the self-focusing effect becomes stronger and the beam width parameter attains a minimium value and maintains it for a long distance. This is because the parameters like density profile and intensity parameter play a vital role for self-focusing of laser beam [13].

In thermal collisionless quantum plasma, the diffraction effect becomes predominant giving rise to increase in beam width and thus, showing an oscillatory behavior of beam width parameter. Further, with the increase in plasma density as a ramp slope, the laser beam focuses quickly with less oscillation amplitude, leads to smaller spot size of laser beam with more oscillations. The laser self-focusing is enhanced more in thermal quantum plasma than in classical regime [14]. However, the magnetic field and plasma density ramp play an important role in the enhancement of self-focusing. This is due to the combined role of magnetic field and density ramp that can reduce the spot size of laser beam efficiently [15]. The self-focusing of the laser beam decreases by increasing laser wavelength, intensity and ripple wave number. It is because of the direct dependence of self-focusing of the beam on

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decentered parameter [16]. Therefore, the phenomenon of self-focusing is obtained by optimizing wavelength and intensity parameters of cosh-Gaussian laser beam. Moreover, the decentered parameter and ramp density profile are sensitive to the self-focusing of laser beam. It is the density ramp that shrinks the spot size of laser beam as it penetrates deeper in to the plasma. Due to which the laser becomes more focussed and can propagate over a long distance without divergence [17]. Further, the proper selection of decentered parameter is important for stronger self-focusing [18]. Gill et al. [19] used the condition for the formation of a dark and bright ring to study the focusing/defocusing of super-Gaussian laser beam in plasma with transverse magnetic field. They included higher order terms of the dielectric function and reported that the inclusion of such terms affects the dependence of beam width parameter on the distance of propagation and consequently substantial increase in self-focusing is observed. This is possible only in case of the dark ring. However, the results contradict in case of a bright ring. Under the ponderomotive self-focusing, the pulse acquires a minimum spot size due to the role of plasma density ramp. The self-focused laser pulse diffracts and focuses periodically because of the mismatch between the channel size and spot size. In such a case the oscillation amplitude of the spot size decreases, while its frequency increases. Further, as the plasma density increases, the laser pulse propagating in plasma under plasma density ramp tends to become more focused. However, if there is no density ramp, the laser pulse is defocused due to the dominance of the diffraction effect [20]. The quantum effect significantly adds self-focusing in plasma as compared to that of classical relativistic case. This is due to the fact that the beam is weaker at high intensity for classical relativistic case than cold quantum case [21].

The initial beam profile of HcosG beam remains invariant during propagation. However, in uniaxial crystals, the initial symmetry and linear polarization of HcosG beam cannot be preserved. In addition the distribution of field of HcosG beam is closely related to the decentered parameter. In this paper, the relativistic self-focusing of Hermite-cosine-Gaussian (HcosG) laser beam in collisionless plasma is investigated. Using WKB and paraxial approximations through the parabolic equation, a mathematical formulation for the beam width parameter in collisionless plasma is obtained from the wave equation. The evolution of beam width parameter with the distance of propagation is presented. It is hereby noticed that the laser self-focusing increases more than predicted by Aggarwal et al. [12]. Moreover, the effect of parameters like decentered parameter, laser intensity and initial plasma density is investigated. We previously studied the self-focusing of Hermite-cosh-Gaussian (HchG) laser beam in plasma under density transition [22] and found that self-focusing occurs under the influence of density ramp and decentered parameter. However, in the present communication, the authors lay emphasis on relativistic self-focusing of HcosG laser beam propagating in underdense plasma which was not done earlier for such a beam to the best of our knowledge. The importance of the present work lies in the fact that an increase in self-focusing length leads to decrease in the minimum spot size of the beam and hence modulates the phenomenon of self-focusing. The computational results in context of plasma density, decentered parameter and laser intensity are discussed and finally a brief conclusion is added. Above all in the present analysis the decentered parameter and laser intensity has good impact on the propagation of HcosG beam in plasma.

## 2. Field distribution of Hermite-cosine-Gaussian (HcosG) beam

Consider the HcosG laser beam propagating in collisionless plasma along $z$-axis with the field distribution in the following form

$$E(x,y,z) = \frac{E_0}{\sqrt{f_1(z)f_2(z)}} H_m\left(\frac{\sqrt{2}x}{r_0 f_1(z)}\right) H_n\left(\frac{\sqrt{2}y}{r_0 f_2(z)}\right) \times \exp\left[-\left(\frac{x^2}{r_0^2 f_1^2(z)} + \frac{y^2}{r_0^2 f_2^2(z)}\right)\right] \times \cos\left(\frac{\Omega_0 x}{f_1(z)}\right)\cos\left(\frac{\Omega_0 y}{f_2(z)}\right) \quad (1)$$

where $H_m$ and $H_n$ are the $m$th and $n$th order Hermite polynomial, respectively, $E_0$ is the constant amplitude of the electric field, $r_0$is the waist width, $\Omega_0$ is the parameter associated with the cosine function, $f_1(z)$ and $f_2(z)$ are the beam width parameters in $x$ and $y$ directions, respectively.

## 3. Non-linear dielectric constant

Consider the propagation of HcosG laser beam in plasma characterized by dielectric constant of the form [23]

$$\varepsilon = \varepsilon_0 + \Phi(EE^*) \quad (2)$$

with $\varepsilon_0 = 1 - \omega_p^2/\omega^2$, $\omega_p^2 = 4\pi n_0 e^2/m$, $m = m_0/\sqrt{1 - v^2/c^2}$ or, $m = m_0\gamma$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the relativistic factor, $m_0$ and $e$ are the rest mass and charge on the electron. Therefore, $\omega_p^2 = \omega_{p0}^2/\gamma$, $\omega_{p0}^2 = 4\pi n_0 e^2/m_0$, $\Phi$ represents the nonlinear part of the dielectric constant, $\omega$ is the angular frequency of laser beam, $\omega_p$ is plasma frequency, $n_0$ is the equilibrium electron density, $R_d$ is the diffraction length and $\xi$ is the normalized propagation distance.

The nonlinear dielectric constant for collisionless plasma can be expressed as [9]

$$\Phi(EE^*) = \frac{\omega_{p0}^2}{\gamma\omega^2}\left[1 - \exp\left(-\frac{3}{4}\frac{m_0\gamma}{M}\alpha EE^*\right)\right] \quad (3)$$

where $\alpha = e^2 M/6m_0^2\gamma^2\omega^2 k_B T$ and $M$ is the mass of scaterrer in the plasma, $k_B$ is the Boltzmann constant and $T$ is the equilibrium plasma temperature.

## 4. Self-focusing

The wave equation governing the propagation of laser beam may be written as

$$\nabla^2 E + \left(\frac{\omega^2}{c^2}\right)\varepsilon E + \nabla\left(\frac{E\nabla\varepsilon}{\varepsilon}\right) = 0 \quad (4)$$

The last term of Eq. (4) on left hand side can be neglected provided that $k^{-2}\nabla^2(\ln\varepsilon) \ll 1$, where '$k$' represents the wave number the laser beam. Thus,

$$\nabla^2 E + \left(\frac{\omega^2}{c^2}\right)\varepsilon E = 0 \quad (5)$$

This equation is solved by employing Wentzel–Kramers–Brillouin (WKB) approximation. Eq. (5) reduces to a parabolic wave equation as:

$$2ik\frac{\partial A}{\partial z} = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{k^2}{\varepsilon_0}\Phi(AA^*)A = 0 \quad (6)$$

To solve Eq. (6) we express $A$ as

$$A(x,y,z) = A_{mn}(x,y,z)\exp[-ikS(x,y,z)] \quad (7)$$

where $k = (\omega/c)\varepsilon_0^{1/2}$ and $A_{mn}$ and $S$ are real functions of $x$, $y$ and $z$. Substituting for $A(x,y,z)$ from Eq. (7) in Eq. (6) and equating real and imaginary parts on both sides of the resulting equation, one obtains: