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Modified function projective lag synchronization of uncertain complex networks with time-varying coupling strength

Shuguo Wang [a,*], Song Zheng [b], Binwu Zhang [a], Haitao Cao [a]

[a] Department of Mathematics and Physics, Changzhou Campus, Hohai University, Changzhou 213022, Jiangsu, PR China
[b] Institute of Applied Mathematics, Zhejiang University of Finance & Economics, Hangzhou 310018, Zhejiang, PR China

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ABSTRACT

In this article, we investigate the issue on modified function projective lag synchronization (MFPLS) of dynamical complex networks with disturbance and unknown parameters, the dynamical network is a complex network model containing uncertainty and coupling delay, the coupling strength is time-varying in the complex networks. Theoretical analysis and numerical simulations about MFPLS of uncertain complex networks are presented, not only unknown parameters of the networks are estimated, but also unknown bounded disturbances can be simultaneously conquered by adaptive laws and adaptive coupling strength obtained from Lyapunov stability theory.

1. Introduction

During the past three decades, complex network [1,2] has been a hot topic in nonlinear science due to its various applications. In reality, a complex network consists of many real complex systems and the links between them. Those systems as the nodes of complex networks can have different meanings in different situations.

One of the interesting phenomena in complex networks is the synchronization, which is an important research subject with the rapidly increasing research. There are many kinds of synchronization, such as complete synchronization [3,4], mismatch synchronization [5], phase synchronization [6,7], anti synchronization [7–11], lag synchronization [12–14], global synchronization [15], projective synchronization [16–18], time scale synchronization [19], combination synchronization [20,21], compound synchronization [22], etc. Function projective synchronization [23,24] is considered as a more important type of dynamical complex networks synchronization, Because the unpredictability of the scaling functions increases the complexity of the systems, Function projective synchronization can be able to improve the security of communication from the standpoint of security. Du [23] investigated function projective synchronization of complex dynamical networks with or without external disturbances using error feedback control scheme. Jin et al. [24] investigated the problem of function projective synchronization for complex networks with switching topology and stochastic effects. A hybrid feedback control method is designed to achieve function projective synchronization for the complex network. Du et al. [25] investigated the function projective synchronization for complex dynamical networks with coupling delay by employing hybrid feedback control method. Recently, a novel type of function projective synchronization method, called modified function projective synchronization [26,27] and there are a few kinds of modified function projective synchronization method have been given, such as modified function projective synchronization [26] and modified hybrid function projective synchronization [27]. Liu et al. [28] introduced synchronization method of adaptive modified projective synchronization with complex scaling matrix for two n-dimensional complex chaotic systems with uncertain complex parameters. Sun et al. [29] investigated the modified projective and modified function projective synchronization between a class of chaotic real nonlinear systems and a class of chaotic complex nonlinear systems. Fu [30] studied the robust modified function projective lag synchronization for different hyperchaotic systems with unknown Parameters. Zheng [31] proposed the concept of the partial switched modified function projective synchronization of complex nonlinear systems with fully unknown parameters. Partial switched synchronization of chaotic systems means that the state variables of the drive system synchronize with partial different state variables of the response system. Gao

* Corresponding author. Tel.: +86 13813563756.
E-mail address: wsg97@163.com (S. Wang).

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et al. [32] studied the modified function projective lag synchronization for chaotic systems with unknown disturbance by using the disturbance observer based control and the linear matrix inequality approach. Wang [33] investigated the modified function projective lag synchronization for two different stochastic chaotic systems using adaptive control method. However, all the above results are concerned with certain systems. As far as we know, the results on modified function projective lag synchronization (MFPLS) of uncertain dynamical complex networks with disturbance and unknown parameters are few, so it is a challenge that how to deal with the MFPLS problem of complex networks which have coupling delay and uncertainty in the coupling matrix.

To the best of our knowledge, the MFLPS of dynamical complex networks with disturbance and unknown parameters has not been reported yet. Hence in this paper we will give a comprehensive study on this topic. The main contributions of this paper are three-fold: (1) This paper deals with MFLPS of dynamical complex networks with disturbance and unknown parameters which are different from the methods used in [26–33]. (2) The dynamical network is a complex network model containing uncertainty and coupling delay. The article considers the uncertainty that appear in the linear coupling matrix, Due to the external interference, the coupling among the nodes may bring about uncertainty. Therefore, it is important to study the uncertain complex networks. (3) The coupling strength is time-varying in the complex networks. These are the three advantages in our work compared with the previous work. Theoretical analysis and numerical simulations about MFPLS of uncertain complex networks are presented, not only unknown parameters of the networks are estimated, but also unknown bounded disturbances can be simultaneously conquered by adaptive laws and adaptive coupling strength obtained from Lyapunov stability theory.

The paper is organized as follows: The network model is introduced followed by some definitions, lemmas, and hypotheses in Section 2. The modified function projective lag synchronization of dynamical complex networks with disturbance and unknown parameters is discussed in Section 3. Simulations are obtained in Section 4. Finally, in Section 5 the various conclusions are discussed.

2. The network model and preliminaries

We consider an n-dimensional complex networks is assumed as

$$\dot{x}_i(t) = g_i(x_i(t)) + G_i(x_i(t))\theta_i + c(t)\sum_{j=1}^{N} b_{ij}(\Gamma + \Delta\Gamma)x_j(t-\tau_1) + \Delta_i \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ denotes the state vector of the ith node, $g_i : R^n \to R^n$ and $G_i : R^n \to R^{n\times m}$ are the known continuous nonlinear function matrices, $\theta_i \in R^{m_i}$ is the unknown constant parameter vector, $\tau_1 \geq 0$ is unknown coupling delay, and $\Delta_i \in R^n$ is the disturbance, $c(t) > 0$ is time-varying coupling strength. $\Gamma = \mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_n)$ is the inner coupling matrix with $\xi_i = 1$ for the ith state, this means that two nodes are coupled via the ith state variable, i.e. Γ determines the variables with which the systems in nodes are coupled, $\Delta\Gamma$ is the uncertainties of inner coupling matrix. $B = (b_{ij})_{N\times N} \in R^{N\times N}$ is the coupling configuration matrix which represents not only the topological structure of the network, but also the weight strength. b_{ij} is defined as follows: if there is a connection from node i to node $j(j \neq i)$, then the coupling $b_{ij} \neq 0$; otherwise, $b_{ij} = 0 (j \neq i)$, and the diagonal elements of matrix B is defined as

$$b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij}, \quad i = 1, 2, \ldots, N, \tag{2}$$

Definition1. Assume that $s(t) \in R^n$ is any smooth dynamics. For dynamical network model with mismatched terms, it is said that the dynamical network model (1) is modified function projective lag synchronization (MFPLS), if there exists a scaling function matrix $M(t)$, such that

$$\lim_{t\to\infty} \left\| e_i(t) \right\| = \lim_{t\to\infty} \left\| x_i(t) - M(t)s(t-\tau) \right\| = 0, \quad i = 1, 2, \ldots, N. \tag{3}$$

where $\tau \geq 0$ is delay.

Let the vector error state be $e_i = x_i(t) - M(t)s(t-\tau)$, $i = 1, 2, \ldots, N$, where $M(t)$ is a n-order real diagonal matrix, i.e. $M(t) = \mathrm{diag}(m_1(t), m_2(t), \ldots, m_n(t))$ and $m_i(t)$ is a continuously bounded differentiable function and $(\dot{m}_1(t), \dot{m}_2(t), \ldots, \dot{m}_n(t))^T = (S_1(m_1, m_2, \ldots, m_n, t), S_2(m_1, m_2, \ldots, m_n, t), \ldots, S_n(m_1, m_2, \ldots, m_n, t))^T$. There exists a stable equilibrium point, a stable periodic orbit or a chaotic attractor in the phase space.

Assumption 1 *([34])*. For any positive constant ε_i, the time-varying disturbances $\Delta_i(t)$ is bounded, i.e. $\left\|\Delta_i(t)\right\| \leq \varepsilon_i$.

Assumption 2 *([35])*. The uncertain matrix $\Delta\Gamma$ is norm bounded and can be given by

$$\Delta\Gamma = F\Phi(t)E$$

where F and E are known constant matrices with appropriate dimensions and uncertain matrix $\Phi(t)$ satisfy $(\Phi(t))^T\Phi(t) \leq I$.

Lemma 1. For any two vectors x and y, a matrix $Q > 0$ with compatible dimensions, one has: $2x^Ty \leq x^TQx + y^TQ^{-1}y$.

Remark 1. The article considers the uncertainty that appear in the linear coupling matrix, Due to the external interference, the coupling among the nodes may bring about uncertainty. Therefore, it is important to study the uncertain complex networks.