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The synchronization of chaotic systems with different dimensions by a robust generalized active control

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ABSTRACT

Active control strategy is a powerful control technique in synchronizing chaotic/hyperchaotic systems. Until now, active control techniques have been employed to synchronize chaotic systems with the same orders. The present study overcomes the limitations of synchronization of chaotic systems of similar dimensions using active control. In this article, the authors investigate the synchronization problem for a drive-response chaotic system with different orders under the effect of both unknown model uncertainties and external disturbance. Based on the Lyapunov stability theory and Routh–Hurwitz criterion, a robust generalized active control approach is proposed and sufficient algebraic conditions are derived to compute a suitable linear controller gain matrix that guarantees the globally exponentially stable synchronization. Two examples are presented to illustrate the main results, namely reduced-order synchronization between the hyperchaotic Lu and the unified chaotic systems and the increased-order synchronization between the unified chaotic and the hyperchaotic Lu systems. There are three main contributions of the present study: (a) generalization of the active control for synchronization of a class of chaotic systems with different orders; (b) a recursive approach is proposed to compute a suitable linear controller gain matrix and (c) reduced (increased) order synchronization under the effect of both unknown model uncertainties and external disturbances. A comparative study has been done with our results and previously published work in terms of synchronization speed and quality. Future applications of the proposed reduced (increased) order synchronization approach are discussed. Finally, numerical simulations are given to verify the effectiveness of the proposed reduced (increased) order active synchronization approach.

1. Introduction

Chaos synchronization has been an area of active research for the successful applications in different scientific fields [1–4]. In this line, various kinds of synchronization control techniques and methods have been developed, such as sliding mode control [5], adaptive control [6], backstepping design [7], phase synchronization [8], projective synchronization [9], active control [10] and the nonlinear active control techniques [11] (among others).

Synchronization of chaotic systems can be classified into two basic categories, namely; synchronization between chaotic systems with the same orders and the synchronization between chaotic systems with different orders. In this direction, two types of chaos synchronization between chaotic systems with different orders have been addressed in the literature, namely; reduced-order synchronization and increased-order synchronization. In reduced/increased order synchronization, the dimension of the drive system is greater (smaller) than that of the response system. Synchronization of chaotic systems with different orders can be found in many natural systems, such as thalamic neurons; in the human brain; synchronization between the heart and lungs, chaotic laser communications and synchronization

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in the cells of paddlefish [12–14] and elsewhere. In this direction, there are a few interesting results in the literature dealing with the synchronization of chaotic systems with different orders. Femat et al. [12] achieved reduced order synchronization between a Chua oscillator and a Duffing system based on the time derivative of system output along the master-slave system vector field. Using the linear errors of corresponding variables and parameters, Ho et al. [13] investigated the reduced-order synchronization between the generalized uncertain Lorenz hyperchaotic and the uncertain Lu chaotic systems. Alsawalha and Noorani [14] proposed the adaptive control to achieve the reduced-order anti-synchronization between the uncertain hyperchaotic Lorenz and the uncertain chaotic Lorenz systems and anti-synchronization between the uncertain hyperchaotic Lu and the uncertain chaotic Chen systems. In 2008, based on the Lyapunov stability theory [15] and using the adaptive control technique, Ge and Yang [16] presented the reduced-order synchronization between the Quantum-CNN with the Lorenz chaotic and regular time function synchronization between the Quantum-CNN and the Chen systems. Recently, the authors [17] investigated the increased-order synchronization and anti-synchronization between the hyperchaotic Lu and the chaotic Lu systems with identification of uncertain parameters. The controller functions and the parameter update laws are derived via an adaptive control method for the uncertain parameters.

Nevertheless, the synchronization methods and control techniques [12–14,16,17] only focused on the chaotic systems in simple or ideal condition. In real engineering applications, chaotic systems are perturbed by unknown model uncertainties and inevitable external disturbance, thus making the control problem more complicated. These results [12–14,16,17] would have been more interesting if it had included the reduced/increased order synchronization behavior under the effect of both unknown model uncertainties and external disturbances.

The active control technique for chaos synchronization was first proposed by Bai et al. [18] based on the linear control theory and further studied by Agiza and Yaseen [19], Ho and Hung [20], Chen [21], Vincent [22], Njah [23], and Ahmad et al. [10], (to name but a few). The active control techniques received considerable attention during the last two decades due to the potential applications to various chaotic systems. These include, nonlinear gyros [20], a permanent magnet reluctance machine [22], an extended Bonhoffer-van der Pol oscillator [23], the electronic circuits which model a third-order "Jerk" equation [24], the RCL-shunted Josephson junction [25], and most recently the spin orbit problem of Enceladus [26]. On theoretical bases, some of the advantages of active control techniques include, the applicability of the technique to certain chaotic systems (either identical or non-identical) regardless of whether the systems contain external excitation or not [27].

From the literature survey, it has been observed that the active control techniques [10,19–27] have been utilized for the synchronization of chaotic systems with the same orders. The stability of the closed-loop system has been established by assigning all the eigenvalues of the coefficient matrix of the error system to the left half of the complex plane. The convergence time can be reduced by choosing large controller gains. In real applications, this may lead to automatic signal saturations. There is no recursive approach [10,19–27] to compute a suitable linear controller gain matrix. In fact, the above notable results [10,19–27] affect each other mutually and need a systematic approach to compute a suitable linear controller gain matrix. In spite of the several reported active control strategies in the literature concerned, some considerable attention is still being paid in addressing the above issues for a better synchronization behavior.

Thus, it is a theoretically as well as practically significant to propose such an active feedback control strategy that synchronizes chaotic systems with different orders under the effect of both unknown model uncertainties and external disturbances with the computation of a suitable linear controller gain matrix.

Motivated by the above discussions, the authors will study, the reduced (increased) order synchronization phenomena under the effect of both unknown model uncertainties and external disturbances. Using the drive-response system synchronization scheme, a generalized active control approach will be proposed and sufficient algebraic conditions will be derived to compute a suitable linear controller gain matrix that would guarantee the globally exponentially stable reduced (increased) order synchronization. Two illustrative examples will be given to verify the robustness and performance of the proposed approach; (a) the reduced order synchronization between the hyperchaotic Lu [28] and the unified chaotic [29] systems and (b) the increased order synchronization between the unified chaotic and the hyperchaotic Lu systems. To the best of the authors' knowledge, the proposed generalized active control technique and the reduced (increased) order synchronization problem under the effect of both unknown model uncertainties and external disturbances have not been addressed in the literature concerned and this has remained an open problem.

There are three main contributions of the present study; (a) generalization of the active control for a class of chaotic systems with different orders (b) a recursive approach is proposed to compute the linear controller gain matrix and (c) the proposed generalized active control approach is robust against the effect of both unknown model uncertainties and external disturbances that increases security of the synchronized systems and improved the reduced/increased order synchronization performance in comparison with notable results. In this paper, we also give some future possible research on the potential applications of the proposed active reduced/increased order synchronization, which of course will be of greater practical value.

The rest of the paper is organized as follows: Section 2 presents the problem statement and a theory for the proposed generalized active control for reduced order and increased order synchronization schemes are given. In Section 3, descriptions of the unified chaotic and the hyperchaotic Lu systems are given and solved the problem of reduced-order synchronization between the unified chaotic and the hyperchaotic Lu systems. Section 4 is devoted to solve the increased order synchronization problem between the hyperchaotic Lu and the unified chaotic systems with some few potential applications of the proposed synchronization approach. At the end, this paper is concluded in Section 5.

2. Theory for the proposed generalized active control

In this section of the paper, the authors briefly describe a theory for the proposed generalized active control to establish the reduced order and increased order synchronization schemes under the effect of both unknown model uncertainties and external disturbances.

Definition 1. A real constant matrix P is said to be positive definite matrix (PDM) if:

(i) All the ordered principal minor determinants being positive.
(ii) P is symmetric and $X^T(t)PX(t) > 0$ for all $X(t) \neq 0$.