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Discrete-time sliding mode control for a quadrotor UAV

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ABSTRACT

This paper mainly addresses the position and attitude tracking control for a small quadrotor UAV via discrete-time sliding mode control (DSMC). Firstly, the linear extrapolation method is used to transform the continuous-time system into discrete-time system. Based on the discrete-time system, the discrete-time flight controllers are designed to perform position and attitude tracking control of the quadrotor UAV. In addition, new conditions are given ensuring the discrete-time system is asymptotically stable. Lastly, based on the kinematic and dynamic model of the quadrotor UAV, extensive simulations are performed to illustrate that the proposed control method has a good performance in terms of stabilization and tracking control.

1. Introduction

In the past few years, the interest in Unmanned Aerial Vehicle (UAV), has been increasing rapidly. Multipurpose, the vehicles have been gaining remarkable capabilities, what's more, they are of importance due to their abilities to replace manned aircraft in mang routine and dangerous missions, and to reduce costs of many aerial operations [1–4], where the several types of missions include that search and rescue missions, wild fire surveillance, power plant inspection, agriculture services, mapping and photographing, and law enforcement.

For the sake of performing autonomous flight, several advanced control methods have been proposed for the quadrotor, such as proportional–integral–differential (PID) control [5], backstepping [6], and evolutive sliding mode control [2,7,8] etc. However, in most of the existing literature of the quadrotor UAV, research efforts on the control methods, have focused primarily in the continuous-time systems and not in the discrete-time systems. In this paper, in order to further investigate the effective discrete-time control method, the discrete-time sliding mode control (DSMC) based on the second order sliding mode technique is studied for performing position and attitude tracking control of the quadrotor UAV.

Recently, the DSMC has been gaining an increasing interest, and many results have been published in [9–13]. In particular, the DSMC has been addressed by resorting to the output feedback approach [9,10]. The DSMC has been investigated by taking the time-varying delays [11]. A reaching condition has been shown to be convenient and effective in [12] to deal with the DSMC for a class of discrete-time systems. In [13], a discrete-time sliding mode controller was proposed for higher order plus delay time processes.

In this paper, the main contribution is to develop a discrete-time sliding mode control approach, which can globally stabilize all states, including those which are indirectly actuated through the nonlinear coupling, for a small quadrotor UAV. Motivated by coupled SMC [8], the DSMC along with a nominal linear coupled sliding manifold designed by incorporating multiple independent states into a sliding manifold is developed for performing the position and attitude tracking control of the quadrotor UAV. The linear coupled sliding manifold is constructed by combining the position and velocity tracking errors of two degrees of freedom in a linear form [8,14,15]. The discrete-time flight controllers are derived via Lyapunov stability theorem, new conditions are given ensuring the discrete-time system is asymptotically stable.

The reminder of this paper is as follows. In Section 2, a kinematic and dynamic model for a small quadrotor UAV is given. In Section 3, the discrete-time flight controllers are designed. The simulations and conclusions are given in Sections 4 and 5, respectively.

2. Quadrotor dynamic model

The kinematic and dynamic model of the quadrotor is utilized in this paper, meanwhile, it is assumed that the configuration structure is rigid and symmetrical, the center of gravity and the body-frame origin coincide, the propellers are rigid and the thrust and drag forces are proportional to the square of the speed of the propeller. The body-frame and the earth-frame are shown in Fig. 1.

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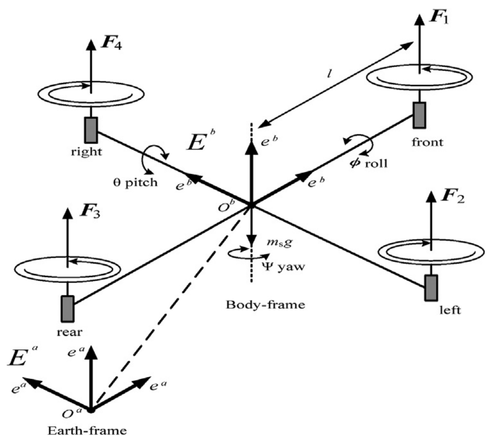

Fig. 1. Quadrotor UAV.

In order to make the controllers designed conveniently, let $[\dot{\phi}, \dot{\theta}, \dot{\psi}] = [p, q, r]$. According to these papers [7,8] the second order nonlinear dynamic model is described by the following equations as:

$$\begin{cases} \ddot{x} = (\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi)\dfrac{u_1}{m} - \dfrac{K_1\dot{x}}{m} \\ \ddot{y} = (\cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi)\dfrac{u_1}{m} - \dfrac{K_2\dot{y}}{m} \\ \ddot{z} = (\cos\varphi\cos\theta)\dfrac{u_1}{m} - g - \dfrac{K_3\dot{z}}{m} \\ \ddot{\varphi} = \dot{\theta}\dot{\psi}\dfrac{I_y - I_z}{I_x} + \dfrac{J_r}{I_x}\dot{\theta}\Omega_r + \dfrac{l}{I_x}u_2 - \dfrac{K_4 l}{I_x}\dot{\varphi} \\ \ddot{\theta} = \dot{\psi}\dot{\varphi}\dfrac{I_z - I_x}{I_y} - \dfrac{J_r}{I_y}\dot{\varphi}\Omega_r + \dfrac{l}{I_y}u_3 - \dfrac{K_5 l}{I_y}\dot{\theta} \\ \ddot{\psi} = \dot{\varphi}\dot{\theta}\dfrac{I_x - I_y}{I_z} + \dfrac{1}{I_z}u_4 - \dfrac{K_6}{I_z}\dot{\psi} \end{cases} \tag{1}$$

where the vector $[x, y, z]'$ denotes the position of the center of the gravity of the aircraft in the earth-frame while the vector $[p, q, r]'$ represents its angular velocity in the body-frame. m denotes the total mass. g denotes the acceleration of gravity. l represents the distance from the center of each rotor to the center of gravity. I_x, I_y and I_z denote the moments of inertia among x, y and z directions, respectively. ϕ, θ and ψ denote the roll, pitch and yaw Euler angles, respectively. K_i (i = 1, 2, 3, 4, 5, 6) are drag coefficients and are positive constants. J_r denotes the moment of inertia. $\Omega_r = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4$, Ω_i (i = 1, 2, 3, 4) denote the ith propeller speed, Ω_r represents the overall speed of propellers.

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -b & 0 & b \\ -b & 0 & b & 0 \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \tag{2}$$

where b is the thrust coefficients, d is the drag coefficient.

3. Flight controller design

This section mainly introduce the discrete-time sliding mode control (DSMC) method that is used to design the discrete-time flight controllers of the quadrotor UAV. In order to make the controllers designed conveniently, the kinematic and dynamic model of the quadrotor UAV are divided into two subsystems: a fully actuated subsystem and an underactuated subsystem, where the fully actuated subsystem involves the second order dynamic equations $\ddot{z}$ and $\ddot{\psi}$, and the underactuated subsystem involves the second order dynamic equations $\ddot{x}$ and $\ddot{\theta}$, $\ddot{y}$ and $\ddot{\phi}$, respectively. Before designing the discrete-time flight controllers, the continuous-time system of the quadrotor (1) is needed to transform into discrete-time system via the linear extrapolation method. The transformed form is given by

$$\begin{cases} x_{k+2} = 2x_{k+1} - x_k + \Delta t^2\left(\cos\varphi_k \sin\theta_k \cos\psi_k + \sin\varphi_k \sin\psi_k\right) u_{1,k}/m \\ -\Delta t \cdot K_1 \left(x_{k+1} - x_k\right)/m \\ y_{k+2} = 2y_{k+1} - y_k + \Delta t^2\left(\cos\varphi_k \sin\theta_k \sin\psi_k - \sin\varphi_k \cos\psi_k\right) u_{1,k}/m \\ -\Delta t \cdot K_2 \left(y_{k+1} - y_k\right)/m \\ z_{k+2} = 2z_{k+1} - z_k + \Delta t^2 \cdot \left(\cos\varphi_k \cos\theta_k \cdot u_{1,k}/m - g\right) - \Delta t \cdot K_3 \left(z_{k+1} - z_k\right)/m \\ \varphi_{k+2} = 2\varphi_{k+1} - \varphi_k + \left(\theta_{k+1} - \theta_k\right)\left(\psi_k - \psi_k\right)\left(I_y - I_z\right)/I_x + \Delta t^2 \cdot u_{2,k} l/I_x \\ +\Delta t \cdot J_r \left(\theta_{k+1} - \theta_k\right) \cdot \Omega_r/I_x - \Delta t \cdot K_4 l \left(\varphi_{k+1} - \varphi_k\right)/I_x \\ \theta_{k+2} = 2\theta_{k+1} - \theta_k + \left(\psi_{k+1} - \psi_k\right)\left(\varphi_{k+1} - \varphi_k\right)\left(I_z - I_x\right)/I_y \\ +\Delta t^2 \cdot u_{3,k} l/I_y - \Delta t \cdot J_r \left(\varphi_{k+1} - \varphi_k\right)\Omega_r/I_y - \Delta t \cdot K_5 l \left(\theta_{k+1} - \theta_k\right)/I_y \\ \psi_{k+2} = 2\psi_{k+1} - \psi_k + \left(\varphi_{k+1} - \varphi_k\right)\left(\theta_{k+1} - \theta_k\right)\left(I_x - I_y\right)/I_x \\ +\Delta t^2 \cdot u_4(k)/I_z - \Delta t \cdot K_6 \left(\psi_{k+1} - \psi_k\right)/I_z \end{cases} \tag{3}$$

where Δt is the sampling time.

3.1. Controller design for fully actuated subsystem

According to the symmetry of a rigid-body quadrotor, $I_x = I_y$ can be obtained. The controllers for the actuated subsystem (z_{k+2}, ψ_{k+2}) are designed by using DSMC. The objective is to ensure that the states z_k and ψ_k converge to their desired equilibrium points z_k^d and ψ_k^d at the instant k, respectively, while assuming z_k^d and ψ_k^d are time invariants.

The sliding manifolds are defined at the instant k

$$s_{z,k} = a_z\left(z_k^d - z_k\right) + \left(\mathrm{d}z_k^d - \mathrm{d}z_k\right) \tag{4}$$

$$s_{\psi,k} = a_\psi\left(\psi_k^d - \psi_k\right) + \left(\mathrm{d}\psi_k^d - \mathrm{d}\psi_k\right) \tag{5}$$

where the coefficients a_z and a_ψ are positive constants; and the change rates of z_k^d and ψ_k^d meet $\mathrm{d}z_k^d = 0$ and $\mathrm{d}\psi_k^d = 0$; $\mathrm{d}z_k = (z_{k+1} - z_k)/\Delta t$, $\mathrm{d}\psi_k = \left(\psi_{k+1} - \psi_k\right)/\Delta t$.

The updated sliding manifolds are derived

$$s_{z,k+1} = a_z\left(z_k^d - z_{k+1}\right) - \mathrm{d}z_{k+1} = \frac{a_z\left(z_k^d - z_{k+1}\right) - \left(z_{k+2} - z_{k+1}\right)}{\Delta t} \tag{6}$$

$$\begin{aligned} s_{\psi,k} &= a_\psi\left(\psi_k^d - \psi_{k+1}\right) - \mathrm{d}\psi_{k+1} \\ &= \frac{a_\psi\left(\psi_k^d - \psi_{k+1}\right) - \left(\psi_{k+2} - \psi_{k+1}\right)}{\Delta t} \end{aligned} \tag{7}$$

The two exponential reaching laws are given by

$$s_{z,k+1} - s_{z,k} = \left(-\eta_1 s_{z,k}\right)\Delta t \tag{8}$$

$$s_{\psi,k+1} - s_{\psi,k} = \left(-\eta_2 s_{\psi,k}\right)\Delta t \tag{9}$$

where η_i $(i = 1, 2) > 0$.

Substituting (4) and (6) for (8), and substituting (5) and (7) for (9), while invoking (3), the discrete-time controllers at the instant k are obtained

$$u_{1,k} = \frac{m}{\cos\varphi_k \cos\theta_k}\left\{\frac{-a_z\mathrm{d}z_k + g + K_3\mathrm{d}z_k}{m + \eta_1 s_{z,k}}\right\} \tag{10}$$

$$u_{4,k} = I_z\left\{\frac{-a_\psi\mathrm{d}\psi_k + K_6\mathrm{d}\psi_k}{I_z + \eta_2 s_{\psi,k}}\right\} \tag{11}$$