Optik 127 (2016) 3803–3808

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Calculating the resonance radius of a dielectric cylinder under illumination by a plane TE-wave

V.V. Kotlyar [a,b], A.A. Kovalev [a,b], D.A. Kozlov [a,b,*]

[a] Image Processing Systems Institute of the Russian Academy of Sciences, Samara, Russia
[b] Samara State Aerospace University, Samara, Russia

ARTICLE INFO

Article history:
Received 15 November 2015
Accepted 10 January 2016

Keywords:
Whispering gallery mode
Dielectric cylinder
Resonance

ABSTRACT

When a dielectric circular cylinder of specific radius is illuminated by a plane wave, a whispering gallery mode (WGM) is generated in the cylinder. This resonance radius of the cylinder can be derived numerically on the assumption of the maximal value of the coefficient in a Bessel series expansion of the E-field amplitude of the TE-wave. Equations composed of cylindrical functions are obtained, allowing the cylinder's resonance radius to be calculated approximately. For instance, based on the equations, one can determine eight initial significant digits of the cylinder resonance radius for a WGM with mode number 26 to be generated. In a cylinder with refractive index 1.59 and resonance radius 3.469239 λ (λ is wavelength), a WGM with mode number 30 is excited. Such a cylinder can generate an external focal spot of size 0.15 λ, with its maximal intensity being 1500 times larger than that of incident light.

1. Introduction

Recent years have seen an extensive study of subwavelength focusing of laser light by means of microparticles, including microbeads and microtubes with their radii being comparable with the wavelength of light [1–5]. For instance, focusing with the aid of multilayer microbeads [1], spherical microparticles [2], and two-layered microspheres [3,4] has been numerically studied. The minimum attained focal spot size at full-width of half-maximum intensity has been found to be FWHM = 0.4 λ [2], with the maximum depth of focus reported being DOF = 20 λ [3], DOF = 22 λ [4] and DOF = 108 λ [5]. Microcylinder-aided focusing has also been reported, including an elliptic [6] and multi-layered cylinder [7]. The minimal focal spot size obtained has been FWHM = 0.46 λ [7]. Resonance focusing of light by means of microspheres was modeled in papers [8,9]. Using a dielectric microsphere in combination with a metal bead, the resonance focusing of light into a spot of size FWHM = 0.25 λ was achieved [8], while a dielectric microsphere produced a resonance focal spot of size FWHM = 0.40 λ [9]. Resonance focusing of a laser TE-wave by means of a polyester microcylinder with refractive index n = 1.59 was studied analytically using a Bessel function series [10]. For a WGM with mode number N = 18, an external focus of size FWHM = 0.22 λ was obtained. In paper [11] it was numerically shown that within several picoseconds while a light pulse travels through a dielectric cylinder with resonance radius, a whispering gallery mode (WGM) is excited, with energy accumulated in the cylinder. Putting it more precisely, two near-surface WGMs propagate in opposite directions, with one traveling clockwise and the other – anticlockwise and forming a standing WGM. Once the ps-pulse has passed through the cylinder, the accumulated WGM's energy decreases with time, with the mode leaking from the cylinder. WGMs in the microresonators have been amply covered in numerous publications (see a review in paper [12]). Rayleigh was the first to propose the WGMs in 1910 [13]. To-date publications handle WGMs in the spherical [14,15], spheroid [16], toroidal [17], disk [18], and other microresonators. Considering that in volume microresonators the Q (or quality) factor ($Q = \lambda/\Delta\lambda$), used to characterize the resonance quality, can experimentally reach an extraordinary value of 10^9 [15], these have been studied most extensively. In an ideal situation of energy losses being only due to radiation, the Q factor can be as high as 10^{57} [12,15] in a dielectric microsphere of diameter 100 μm. As far as cylindrical microresonators are concerned, these have a lower Q factor, and, thus, publications dealing with WGM in cylindrical microcylinders are scarce. For example, spiral WGMs in a conventional single-mode optical fiber were discussed in [19].

In this work, we study WGMs that are excited when a circular microcylinder of specific radius is illuminated by a plane monochromatic wave. The resonance radius of interest can be derived numerically on the assumption of the maximal modulus of the coefficient of a Bessel series expansion of the electric field

* Corresponding author at: Image Processing Systems Institute of the Russian Academy of Sciences, Samara, Russia. Tel.: +7 9272970674.
E-mail address: kozlov.dmitry.a@gmail.com (D.A. Kozlov).

http://dx.doi.org/10.1016/j.ijleo.2016.01.058
0030-4026/

strength amplitude of a TE-wave. The equations in cylindrical coordinates derived enable us to obtain an approximate estimate of the microcylinder's resonance radius. Several simple approximate relations to calculate the microcylinder's resonance radius with correct two-to-three initial significant digits are given. Resonance radii for WGMs with mode numbers varying from 3 to 30 are derived. Relations for the maximum on-axis intensity just above cylinder surface, width and depth of focus as functions of the WGM number or a microcylinder's resonance radius are estimated.

2. Calculating the microcylinder resonance radius by deriving the maximum coefficient of the field expansion into cylindrical functions

An analytic solution of a plane-wave diffraction problem by an infinite dielectric circular cylinder was proposed in the book [20]. The solution to the Helmholtz equation for the E-field strength of a *E*-wave in the microcylinder has been found by separation of variables in polar coordinates (r, ϕ) is given by a Bessel series expansion:

$$E_y(r,\phi)=\sum_j i^j b_j J_j(knr)e^{ij\phi}, \tag{1}$$

where k is the wavenumber of incident light, n is the refractive index of the microcylinder, and the coefficient b_j is defined as

$$b_j=\frac{J_j(z)H_j^{(1)\prime}(z)-J_j'(z)H_j^{(1)}(z)}{J_j(nz)H_j^{(1)\prime}(z)-nJ_j'(nz)H_j^{(1)}(z)}, \tag{2}$$

where $z=kR$, R is the cylinder radius, $J_m(x)$, $H_m^{(1)}=J_m(x)+iY_m(x)$ are the Bessel and Hankel functions, and $Y_m(x)$ is the Neumann function. Primes at the right upper corners of the functions mean differentiation of the functions with respect to their arguments. The maximal absolute value of coefficient (2) can be derived by simplifying the relation based on well-known property of cylindrical functions given by a Wronskian:

$$b_m=\frac{J_m(z)H_m^{(1)\prime}(z)-J_m'(z)H_m^{(1)}}{J_m(nz)H_m^{(1)\prime}(z)-nJ_m'(nz)H_m^{(1)}(z)}=\frac{2i}{\pi z}\frac{1}{J_m(nz)H_m^{(1)\prime}-nJ_m'(nz)H_m^{(1)}(z)}. \tag{3}$$

Mathematically speaking, the zero-valued denominator in Eq. (2) or Eq. (3) leads to the zero-valued determinant of a homogeneous linear system of second-order algebraic equations for deriving the coefficient b_m in Eq. (1), given that light is not incident on the cylinder.

Fig. 1 depicts the modulus of coefficient (3) at $m=18$ as a function of the cylinder's radius R in terms of wavelengths: Fig. 1(a) shows the radius in the range $0\le R\le 10\lambda$ and Fig. 1(b) depicts a magnified fragment of the curve in the range $2.1\lambda\le R\le 2.3\lambda$.

From Fig. 1(b) the coefficient is seen to reach its maximum value of 12 at $R_{18}=2.174987\ \lambda$, whereas the maximum's FWHM width is $\Delta R=0.003\ \lambda$. Thus, given the same wavelength, which a change in radius as small as 0.14% the resonance will not occur any more, which is equivalent to the absence of WGM [11].

Fig. 2 depicts a different $|b_m|$ in Eq. (3) at $m=21$ as a function of cylinder's radius R in terms of wavelengths for two different refractive indices of the cylinder: silica, $n=1.46$ (Fig. 2(a)) and polyester $n=1.59$ (Fig. 2(b)). It is the first and highest maximum that corresponds to the microcylinder's resonance radius: $R_{21}=2.707071\ \lambda$ (Fig. 2(a)) and $R_{21}=2.502264\ \lambda$ (Fig. 2(b)). Note that for all first maxima of the coefficient $|b_m|$ there fulfills the following relation:

$$\frac{y_{m,1}}{2\pi n}<\frac{R_m}{\lambda}<\frac{j_{m,1}}{2\pi n},\ Y_m(y_{m,1})=0, J_m(j_{m,1})=0, \tag{4}$$

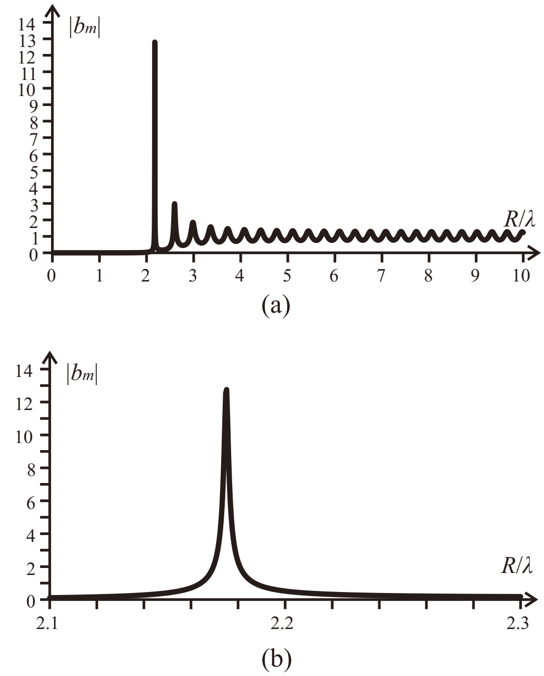

Fig. 1. *R*-dependence of $|b_m|$ at $m=18$, $0\lambda\le R\le 10\lambda$ (a) and $2.1\lambda\le R\le 2.3\lambda$ (b).

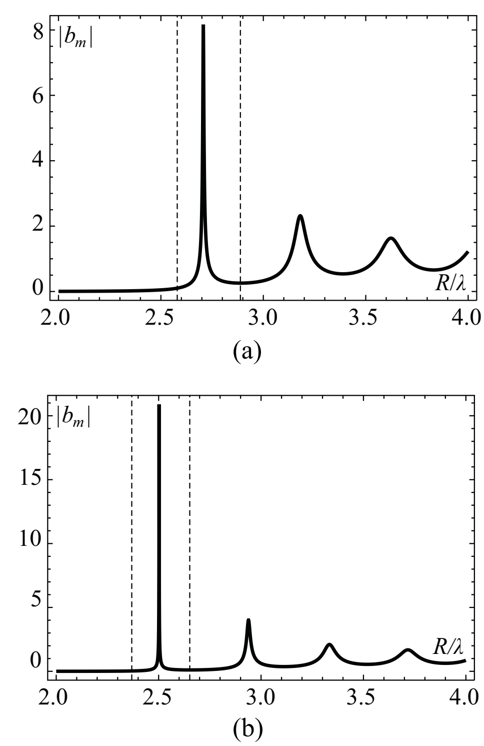

Fig. 2. Coefficient (4) vs. radius of a dielectric cylinder with the refractive indices (a) 1.46 (silica) and 1.59 (polyester).

where R_m is the first maximum's radius for the m-th mode, $y_{m,1}$ is the first zero of the m-th order Bessel (Neumann) function of the second kind, and $j_{1,m}$ is the first zero of the m-th order Bessel function of first kind. In Fig. 2(a) and (b) the boundary values of inequality (4) are marked with vertical dashed lines.

Fig. 2 also suggests that as the microcylinder's refractive index increases, the modulus of the coefficient also increases, while the resonance peak becomes narrower. Considering that in Figs. 1 and 2,