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Angle of arrival fluctuations of optical waves considering finite turbulence outer scale under anisotropic turbulence

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ABSTRACT

In this work, new analytic expressions for the variance of angle of arrival (AOA) fluctuations are derived based on the Rytov theory for optical plane and spherical waves propagating through weak anisotropic non-Kolmogorov atmospheric turbulence. In the derivations, the concept of anisotropy at different turbulence cells is considered and the finite turbulence outer scale is included. Deviations from the classic 11/3 spectral power law behavior for Kolmogorov turbulence are also allowed by assuming spectral power law value variations between 3 and 4. The derived analytic expressions can reduce correctly to the previously published results for optical plane and spherical waves propagating through weak anisotropic non-Kolmogorov turbulence in the special case of the finite turbulence inner and outer scales equaling separately zero and infinite. Calculations are performed to analyze the influences of anisotropic non-Kolmogorov turbulence on the derived analytic results.

1. Introduction

For long-range photoelectric imaging system, the objects in the images will be distorted due to the random atmosphere refractive-index fluctuations. They are caused by the optical wave phase distortion and can be described by the AOA fluctuations of optical waves. Investigations of AOA fluctuations of optical waves play an important role in the image restoration and the design of photoelectric imaging system. For a long time, the researches on the AOA fluctuations are focused on the isotropic atmosphere turbulence and several theoretical models of the variance of AOA fluctuations have been derived [1–6]. These researches mainly focused on the isotropic atmospheric turbulence. However, experimental and theoretical results have shown that the atmospheric turbulence can also be anisotropic [7–20]. Grechko et al. [9] reported a strong anisotropy in the middle atmosphere from experimental observations of star scintillation. Biferale et al. [10] detected the information about anisotropic turbulence in the boundary layer by using two probes with two different geometries (horizontally and vertically). Belen'kii et al. [11,12] experimentally observed anisotropy of the statistics of wavefront tilt. They observed a horizontal outer scale bigger than the vertical one and the horizontal tilt variance is consistently greater than the vertical one. Also, evidence of anisotropy in the stratosphere has been reported in [13], where authors used a power spectrum with two components: anisotropic and isotropic, and the validity of the spectrum was verified by balloon-borne experiments. Anisotropy is usually present at high altitude, above the atmospheric boundary layer, which extends to about 2 km in altitude and it is more evident for large turbulence cells or eddies [19]. Anisotropy can be present also at few meters above the ground [7]. The theoretical investigations of optical waves' propagation under anisotropic non-Kolmogorov turbulence attract more and more researchers [19–25].

Recently, based on the anisotropic generalized non-Kolmogorov atmosphere turbulence spectrum [17,19] which adopts the circular symmetric assumption of turbulence cells or eddies in the orthogonal *xy*-plane throughout the path, the analytic expressions of the variance of AOA fluctuations have been derived theoretically for optical plane and spherical waves propagating through weak anisotropic non-Kolmogorov turbulence [25]. But the atmosphere turbulence refractive-index fluctuations spectrum adopted in [25] is only valid in inertial range. The finite turbulence inner and outer scales have not been considered in the derived results, also the asymmetry property of turbulence cells or eddies in the orthogonal *xy*-plane throughout the path has not been included. Toselli [21] introduced the finite turbulence inner and outer scales into the atmosphere turbulence refractive-index fluctuations spectrum, and the concept of anisotropy

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at different scales has also been introduced through an effective anisotropic parameter which was defined for two specific cases of linear and parabolic anisotropic laws. This spectrum has been adopted to investigate the spectral density, the spectral degree of coherence, and the spectral degree of polarization of the classic electromagnetic Gaussian Schell-model beams [23].

In this study, taking the finite turbulence inner and outer scales and the effective anisotropic factor which parameterizes the anisotropy at different scales into considerations, the new analytic expressions for the variance of AOA fluctuations will be derived for optical plane and spherical waves propagating through weak anisotropic non-Kolmogorov turbulence. First, the general anisotropic non-Kolmogorov turbulence refractive-index fluctuations spectrum reported by Toselli [21] will be introduced. Second, with the Rytov theory and taking the effective anisotropic factor and finite turbulence inner and outer scales into consideration, new analytic expressions for the variance of AOA fluctuations will be derived. Last, calculations will be performed to analyze the derived results.

2. Anisotropic turbulence spectrum with finite turbulence inner and outer scales

The anisotropic non-Kolmogorov power spectrum reported in [21] includes the turbulence inner and outer scale effects by using a generalized von Karman model. It assumed that the anisotropy exists only along the direction of propagation (z direction) of the beam and this anisotropy was accounted via an effective anisotropy factor ς_{eff} as discussed in [21]. The spectrum takes the form as [21]:

$$\Phi_n(\kappa,\alpha,\varsigma) = A(\alpha)\cdot \hat{C}_n^2 \cdot \varsigma_{eff}^2 \cdot \left(\varsigma_{eff}^2\kappa_{xy}^2+\kappa_z^2+\kappa_0^2\right)^{-\frac{\alpha}{2}} \exp\left(-\frac{\varsigma_{eff}^2\kappa_{xy}^2+\kappa_z^2}{\kappa_m^2}\right), \quad \left(\kappa>0,\quad 3<\alpha<4\right). \tag{1}$$

in which, κ is the wavenumber related to the turbulence cell size, $\kappa=\sqrt{\varsigma_{eff}^2\left(\kappa_x^2+\kappa_y^2\right)+\kappa_z^2}=\sqrt{\varsigma_{eff}^2\kappa_{xy}^2+\kappa_z^2}$, κ_x, κ_y, and κ_z are the components of κ in the x, y, and z directions. α is the general spectral power law, $\kappa_0=2\pi/L_0$, $\kappa_m=c(\alpha)/l_0$, l_0 is the turbulence inner scale, and L_0 is the turbulence outer scale. $\hat{C}_n^2=\gamma C_n^2$ is the generalized structure parameter with unit $\left[\mathrm{m}^{3-\alpha}\right]$, and γ is a dimensional constant with unit $\left[\mathrm{m}^{11/3-\alpha}\right]$. For Kolmogorov power law $(\alpha=11/3)$, the generalized structure parameter reduces to the structure parameter C_n^2 with unit $\left[\mathrm{m}^{-2/3}\right]$. $A(\alpha)$ is a constant which maintains consistency between the refractive index structure function and its power spectrum [21]:

$$A(\alpha)=\frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left[\frac{\alpha\pi}{2}\right],\ c(\alpha)=\left\{\pi A(\alpha)\Gamma\left(\frac{3}{2}-\frac{\alpha}{2}\right)\left(\frac{3-\alpha}{3}\right)\right\}^{\frac{1}{\alpha-5}}. \tag{2}$$

$\Gamma(\cdot)$ is the gamma function. In the following analyses, the propagation path is along z direction, at this time, κ_z can be ignored by invoking the Markov approximation which implies that the index of refraction is delta-correlated at any pair of points located along the direction of propagation.

3. Variance of AOA fluctuations in anisotropic non-Kolmogorov turbulence

Considering the relation between the variance and covariance, the variance of AOA fluctuations can be derived from the covariance of AOA fluctuations. According to the Wiener–Kinchin theorem, the spatial covariance function of the AOA fluctuations $C_\theta(\cdot)$ can be expressed as [26]:

$$C_\theta\left(\rho,\beta\right)=\pi k^{-2}\int_0^\infty \kappa^3 W_\phi(\kappa)G_D(\kappa)\left[J_0(\rho\kappa)-\cos\left(2\beta\right)J_2(\rho\kappa)\right]d\kappa. \tag{4}$$

where ρ represents the geometrical separation between points in the plane transverse to the direction of propagation, β is the angle between the baseline (z-axis) and the AOA observation axis, $k=2\pi/\lambda$ and λ denotes the optical wavelength. $J_0(\rho\kappa)$ denotes the zero order Bessel function. $W_\phi(\kappa)$ is the wave-front phase power spectrum. $G_D(\kappa)$ represents the point spread function of the receiver aperture [27]:

$$G_D(\kappa)\approx\exp\left(-\frac{b^2D^2\kappa^2}{4}\right),\quad b=0.52. \tag{5}$$

Using the relation between the variance and covariance, when ρ and β in Eq. (9) equal to zero, the variance of the AOA fluctuations can be obtained as follows:

$$\sigma^2=C_\theta\left(\rho=0,\beta=0\right)=\pi k^{-2}\int_0^\infty \kappa^3 W_\varphi(\kappa)G_D(\kappa)d\kappa. \tag{6}$$

In view of the Rytov approximation theory, the wave-front phase power spectra for plane and spherical waves are given by

$$W_{\phi(pl)}(\kappa)=2\pi k^2\int_0^L \Phi_n\left(\kappa,\alpha,\varsigma_{eff}\right)\cos^2\left(\frac{\kappa^2 z}{2k}\right)dz, \tag{7}$$

$$W_{\phi(sp)}(\kappa)=2\pi k^2\int_0^L \Phi_n\left(\kappa,\alpha,\varsigma_{eff}\right)\left(\frac{z}{L}\right)^2\cos^2\left[\frac{\kappa^2 z(L-z)}{2kL}\right]dz. \tag{8}$$