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# Evolution of the coherent state via a new time evolution operator

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ABSTRACT

Inspired by the dynamical Casimir effect, we construct a new time evolution operator (NTEO). We study the time evolution of the coherent state (TECS) via the NTEO by the technique of integration within an ordered product (IWOP) of operators. We investigate the quantum statistical properties of it, both analytically and numerically, by evaluating its Q-function, second-order correlation function, photon-number distribution and Wigner function. Furthermore, the fidelity between the TECS and the classical coherent state is also given. It is interesting to find that smaller modulation depth can lead to more chance of sub-Poissonian statistics in some cases. It is also shown that the new time evolution operator not only has squeezing effect, but also has rotating effect.



## 1. Introduction

Dynamical Casimir Effect (DCE) was first studied by Yablonovitch [1] and Schwinger [2]. In recent years the DCE has been an important subject in theoretical studies [3–5]. DCE is a generation of photons from vacuum state in macroscopic systems by the motion of uncharged boundaries [6,7] or a index of refraction [8].

During the past years, there are many different concrete experimental proposals for the DCE were given. In Ref. [9], DCE was modeled in a superconducting stripline waveguide terminated by a superconducting quantum interference device with a rapidly varying magnetic flux. DCE can also be realized by a closed cavity with moving walls [10]. The numbers of photons inside this cavity can be increased significantly for periodical motion of the wall(s) under certain resonance conditions [11,12]. The Hamiltonian describing this effect without dissipation is ($\hbar = 1$) [13]

$$H = \omega_t n - i\chi_t a^2 + i\chi_t a^{\dagger 2}, \tag{1}$$

where $a$ and $a^\dagger$ are the cavity annihilation and creation operators, $n \equiv a^\dagger a$ is the photon number operator, $\omega_t \equiv \omega_0 [1 + \varepsilon \sin(\eta t)]$ is the cavity instantaneous eigen frequency, $\varepsilon$ is a modulation depth, $\eta$ is the modulation frequency which close to the parametric resonance frequency $\eta = 2(1 + x)$, and $\chi_t = (4\omega_t)^{-1} \mathrm{d}\omega/\mathrm{d}t$.

The Hamiltonian of the dynamical Casimir cavity (DCC) in Eq. (1) can also be realized by the experiment [14,15]. It can transform the initial vacuum state into the squeezed vacuum state so that only even numbers of quanta can be generated with nonzero probabilities. Inspired by Eq. (1), an interesting question thus naturally arise: can we find a NTEO which is described by Eq. (1) and what nonclassical properties are? The answer is affirmative.

In this passage, we shall study the TECS in the which described by the Hamiltonian in Eq. (1) and its quantum statistical properties. This paper is organized as follows. In Section 2, we derive the explicit expression of the TECS by the technique of IWOP. In Section 3, based on the new expression, we investigate the quantum statistics of it by valuating its Q-function, second-order correlation function, photon-number distribution and Wigner function. The fidelity between the TECS and the classical coherent state is discussed in Section 4. Our main results are summarized in Section 5.

## 2. Explicit expression of the TECS

In this section, we shall study how an initial coherent state $|\alpha\rangle$ evolves via the NTEO based on the technique of IWOP of operators. We construct the NTEO as

$$|\alpha\rangle_t = U(t)|\alpha\rangle = \exp\left[-\left(i\omega_t n + \chi_t a^2 - \chi_t a^{\dagger 2}\right)t\right]|\alpha\rangle. \tag{2}$$

It should be noted that the NTEO $U(t)$ in Eq. (2) is different from the time evolution operator for the Hamiltonian in the Schrödinger representation.

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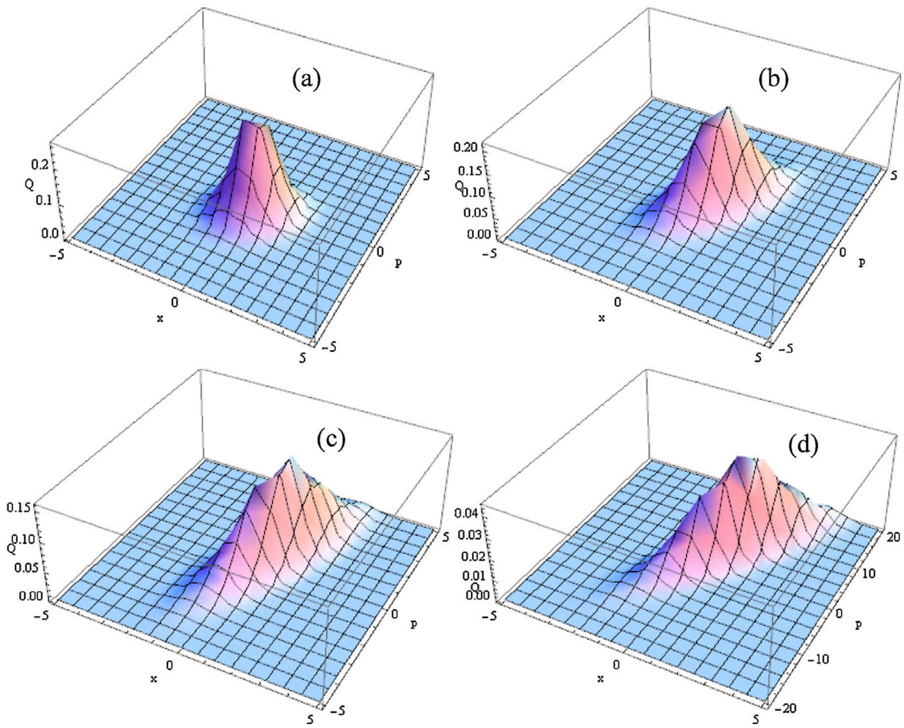


**Fig. 1.** Q-function distributions for $|\alpha\rangle_t$ with $\omega_0 = 1, \quad \eta = 2.01, \quad \alpha = 1/2 + \left(1/2\right) i, \quad t = 2$ for different modulation depth (a) $\varepsilon = 0.2$ ; (b) $\varepsilon = 0.4$ ; (c) $\varepsilon = 0.6$ ; (d) $\varepsilon = 0.8$.

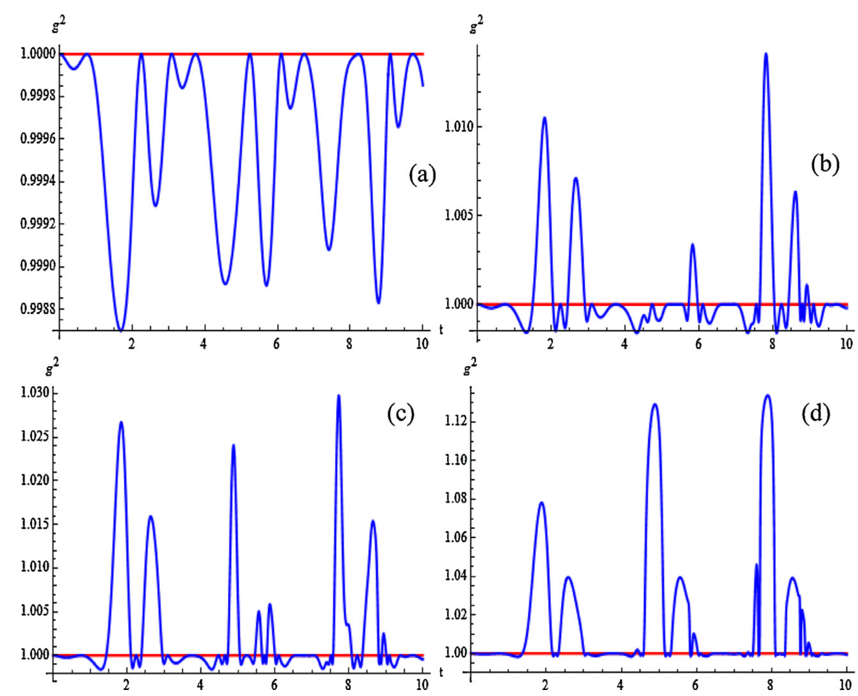


**Fig. 2.** The second-order correlation function $g^2$ for the TECS as a function of the parameter $t$ with $\omega_0 = 1, \quad \eta = 2.1, \quad \alpha = 1/2 + \left(1/2\right) i$ for different parameter of the modulation depth $\varepsilon$:(a) 0.2; (b) 0.4;(c) 0.45;(d) 0.55.

To derive the explicit expression of Eq. (2), we shall use the concise operator identity [16]

$$\exp\left(fa^\dagger a + ga^{\dagger 2} + ka^2\right)$$
$$= e^{-f/2} e^{ga^{\dagger 2}/D\coth D - f} e^{\left(a^\dagger a + 1/2\right)\ln D\sec hD/D - f\tanh D} e^{ka^2/D\coth D - f}, \quad (3)$$

where

$$D = \sqrt{f^2 - 4\,\mathrm{kg}}. \quad (4)$$

It follows

$$U(t) = e^{(i/2)\omega_t t} exp\left(D_1 a^{\dagger 2}\right) exp\left[\left(a^\dagger a + \frac{1}{2}\right) lnD_2\right] exp\left(-D_1 a^2\right) \quad (5)$$

where

$$D_0 = \sqrt{4\chi_t^2 - \omega_t^2}, \qquad D_1 = \frac{\chi_t}{D_0 \coth D_0 t + i\omega_t}, \qquad D_2 = \frac{D_0 \sec hD_0 t}{D_0 + i\omega_t \tanh D_0 t}. \quad (6)$$