Optik 127 (2016) 3895–3898

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Tunable Faraday effect in one-dimensional photonic crystals doped by plasma

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ARTICLE INFO

Article history:
Received 9 September 2015
Accepted 28 December 2015

Keywords:
One-dimensional photonic crystal
Photonic band gap
Faraday rotation

ABSTRACT

In this paper, a four by four transfer matrix method has been used to investigate the optical and magneto-optical properties of a one-dimensional photonic crystal (1D-PC) doped by plasma. We show theoretically that there are a tunable resonance splitting form and a magnetically induced birefringence in a 1D-PC doped by plasma. The Faraday rotation can be seen clearly in this kind of structures. It is also figured out that the magnitude and the frequency of maximum Faraday rotation affected by the variation of the plasma density and the external magnetic field. This prospective advantage of the structure is useful for tunable filters in millimeter-wave region.

1. Introduction

For the last decade, photonic crystals (PCs) have been the object of intensive studies due to their outstanding optical properties that are very promising for the numerous integrated-optics applications [1–3]. These structures exhibit a band gap over which an electromagnetic wave cannot propagate. When the periodicity is broken by introducing a defect into a conventional PC, a localized defect mode can be seen inside the band gap owing to the change of the interference behavior of electromagnetic (EM) wave. When one of the components of the photonic crystals (PCs) is a magnetic material, such as ferrite or ferromagnet, a magnetic defect is introduced into the PC, which remarkably enhances the magneto-optical effect that occurs in the magnetic films. The enhancement is ascribed to the localization of the electromagnetic (EM) wave in the magnetic layer, which can be attributed to the multi-reflection of the wave in the multilayer structures of the MPCs [4–6]. So far, most of the wavelength regions or resonance channels are within the range of light. However, according to the scale invariant of photonic crystals [7], if the thickness of the layers are compressed or expanded for the same scale parameter, *l*, the peak frequency of all the above calculations will increase or decrease for the same scale parameter l [8]. Plasma photonic crystal (PPC), which contains more particular characteristics than the conventional PCs, has a new introduction in tunable photonic ban gap (PBG) materials [9,10]. Plasma is a kind of dispersive medium which can be easily magnetized, so these structures can be tunable by plasma parameters and the external magnetic field. In the presence of the external magnetic field, magneto-optical effects can be occurred in plasma medium. A well-known example of these magneto-optical effects is Faraday rotation related to circular birefringence [11]. Several theoretical and experimental studies on PPCs at the millimeter wave region have been conducted [12–16]. All of these works focused on the PBG characteristics of the 1D-PPC composed of alternating thin plasma and other materials. Zhang et al.[17] and Qi et al. [18] studied the optical properties of 1D-PPC with layer defects by the FDTD method. They showed that the PPC with defect modes, which emerge in the PBGs, can be utilized as narrow filters. Kong et al. [19] and Whang et al. [20] investigated the dependence of defect mode in 1D-PC doped by unmagnetized and magnetized plasma defect layer on plasma parameters. In the references published previously, we have found no studies of faraday effect in PC with plasma defect layer. However it can be seen that transmission and faraday rotation can be easily tuned by the aid of this structure. In this connection, we applied the four by four transfer matrix method to investigate the transmission spectrum and the Faraday rotation angle in a 1D-PC structure doped by magnetized plasma. We have investigated the magnetically induced birefringence of plasma and its dielectric tensor dispersion. Results show a tunable resonance splitting form and a magnetically induced birefringence due to the coherent interference between a right and a left circularly polarized mode. Also it has been shown that the Faraday rotation altered with the variation of the plasma density and the external magnetic field. This provides a unique opportunity to probe the characteristics of plasma via optical routes. The organization of the paper is as follows. The 4×4 transfer matrix approach for 1D-PC doped by magnetized plasma

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http://dx.doi.org/10.1016/j.ijleo.2015.12.123

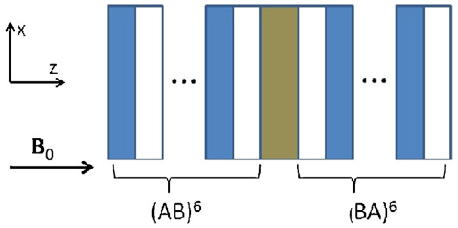

Fig. 1. Schematic view of 1D-PC structure doped by magnetized plasma. An electromagnetic wave radiation is incident normally on the first surface of the system. The z-axis is parallel to the external field and electromagnetic wave vector.

is deduced in Section 2. The dependence of transmission and Faraday rotation on plasma density and the external magnetic field are studied in Section 3. Finally conclusions are given in Section 4.

2. Theory

Consider a basic example of 1DPC structure as $(AB)^N APA (AB)^N$ with repetition number N, shown in Fig. 1. Here A and B are higher and lower refractive index materials, where alternately placed in the multilayer stack of dielectric materials, P is a layer of magnetized plasma. The optical thickness of layers A, B and P are defined as $d_a = \lambda_0/4n_a$, $d_b = \lambda_0/4n_b$ and $d_p = \lambda_0/n_b$, respectively. Where λ_0 is the mid-gap wavelength of PC in normal incidence. The 1DPC structure arranged in the z direction and the external magnetic field $\vec{B}_0$ is along with the wave vector of normally incident electromagnetic wave. For an electromagnetic wave with angular frequency ω its fundamental equations are given by Maxwell equations:

$$\vec{\nabla} \times \vec{E}(\vec{r}, t) = i\omega\mu_0 \vec{H}(\vec{r}, t), \tag{1}$$

$$\vec{\nabla} \times \vec{H}(\vec{r}, t) = -i\omega\varepsilon_0 \hat{\varepsilon} \vec{E}(\vec{r}, t), \tag{2}$$

where $\hat{\varepsilon}$ is the relative permittivity tensor that for magnetized plasma as an anisotropic medium has this form [21]

$$\widehat{\varepsilon_p} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \tag{3}$$

where

$$\varepsilon_1 = 1 - \frac{\omega_p^2(\omega + i\nu)}{\omega[(\omega + i\nu)^2 - \omega_c^2]}, \tag{4}$$

$$\varepsilon_2 = \frac{-\omega_p^2 \omega_c}{\omega[(\omega + i\nu)^2 - \omega_c^2]}, \tag{5}$$

$$\varepsilon_3 = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}. \tag{6}$$

Here ω is the incident wave frequency, $\omega_p = \sqrt{n_e e^2/m_e \varepsilon_0}$ is the electron plasma frequency as a function of plasma density n_e, $\omega_c = eB_0/m_e$ is the cyclotron frequency of the electron and ν is the collision frequency. When a linearly polarized wave propagates parallel to the z-axis in the magnetized plasma layer, the wave equation provides two different values for refractive index as:

$$\varepsilon_+ = 1 - \frac{\omega_p^2}{\omega - \omega_c + i2\nu}, \tag{7}$$

$$\varepsilon_- = 1 - \frac{\omega_p^2}{\omega + \omega_c + i2\nu}. \tag{8}$$

Thus the plane of polarization rotates in the direction of propagation as a function of distance, plasma density and the magnetic field strength. Calculation of Faraday rotation and transmission were performed by employing four by four transfer matrix approach. This has been used and proved as an effective technique to handle the optical properties of magnetic materials. In this method, the optical field inside each layer is given as the sum of four normal modes: right and left circular polarized waves propagating in both directions along 1DPC normal. Then a set of 4 × 4 matrices is calculated, each matrix corresponds to a layer of the structure and determines the values of the optical fields on the layer boundaries. The amplitude of forward (E_x^f, E_y^f) and backward (E_x^b, E_y^b) propagating modes at the front and $(z = z_0)$ and rear $(z = z_N)$ of the stack are related by $\vec{E}(N) = \widehat{M}\vec{E}_0$ Where $\vec{E} = (E_x^f, E_y^f, E_x^b, E_y^b)$ and the 4 × 4 matrix representing the whole structure is given by:

$$\widehat{M} = [D^{(0)}]^{-1} \left(\prod_{n=1}^{12} S^{(n)}\right) S^P \left(\prod_{m=1}^{12} S^{(m)}\right) D^{(0)}. \tag{9}$$

where D and S are dynamic and block diagonal medium matrix that are given by [22]

$$\widehat{S} = \begin{pmatrix} \cos\beta_+^{(n)} & iN_+^{(n)-1}\sin\beta_+^{(n)} & 0 & 0 \\ iN_+^{(n)}\sin\beta_+^{(n)} & \cos\beta_+^{(n)} & 0 & 0 \\ 0 & 0 & \cos\beta_-^{(n)} & iN_-^{(n)-1}\sin\beta_-^{(n)} \\ 0 & 0 & iN_-^{(n)}\sin\beta_-^{(n)} & \cos\beta_-^{(n)} \end{pmatrix}, \tag{10}$$

and

$$\widehat{D} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ N_+^{(n)} & -N_+^{(n)} & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & N_-^{(n)} & -N_-^{(n)} \end{pmatrix}. \tag{11}$$

Here $\beta_\pm^{(n)} = \frac{\omega}{c} N_\pm^{(n)} d_n$, $N_\pm^{(n)}$ and d_n denote the thickness and refractive index of right and left circularly polarized wave in n-th layer. It should be noted that for dielectric layers $N_+^{(n)} = N_-^{(n)}$. Once the matrix $\widehat{M}$ is known the transmission of each circular polarization $(t_\pm, r_\pm)$ can be easily found from $t_+ = (M_{11})^{-1}$, $t_- = (M_{33})^{-1}$. The total transmission coefficient is $T = |t_+ + t_-|^2/4$. By the aid of these expressions the Faraday rotation angle is calculated by $\theta_f = \frac{1}{2} arg(\frac{M_{11}}{M_{33}})$.

3. Numerical results and discussions

In our numerical calculations, the considered multilayer film is composed of SiO_2 with $n_a = 2$ and air with $n_b = 1$ for dielectric layers. We choose the cold, steady state and collisionless plasma with $n_e = 1.49 \times 10^{18}\ \mathrm{m}^{-3}$ ($\omega_p = 11$ GHz), $\nu_e = 0$ for electromagnetic waves with the basic frequency $f_0 = c/2(n_a d_a + n_b d_b) = 2$ GHz or wavelength $\lambda_0 = 2(n_a d_a + n_b d_b) = 0.15$ m, and the repetition number of 1DPC structure is taken to be 6. At first we discuss about the transmission spectrum of a linearly polarized wave through the structure. It is shown in Fig. 2(a) that there exist a very sharp peak of transmission of about 100% in the stop band. This peak is due to the defect mode and physically, it is analogous to an impurity level for semiconductors. Fig. 2(a) also shows that the frequency of defect mode changes for different values of the plasma density. The refractive index of the plasma, as defect layer, changes with the increase of the plasma density. It was shown that different values of optical width of the defect layer, change the resonant wavelength as well as the resonant electromagnetic wave frequency. In fact, every one of these peaks are consisting of two merged peaks, since ε_+ merely differs slightly from ε_- for frequencies in band. However by increasing the magnitude of B_0 to appropriate values, these two merged peaks will be splited (Fig. 2(b)). By increasing the external magnetic field,