Optik 127 (2016) 3943–3949

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Enhanced dynamic error spectrum for estimation performance evaluation in target tracking

Wei-Shi Peng [a,b,*], Yang-Wang Fang [a], Dong Chai [a]

[a] School of Aeronautics and Astronautics Engineering, Air Force Engineering University, Xi'an, Shaanxi 710038, China
[b] School of Equipment Engineering, Armed Police Force Engineering University, Xi'an, Shaanxi 710086, China

ARTICLE INFO

Article history:
Received 27 November 2015
Accepted 12 January 2016

Keywords:
Performance evaluation
Estimation
Error spectrum
Dynamic error spectrum
Target tracking

ABSTRACT

The error spectrum is a comprehensive metric for evaluation of estimation performance, which is an aggregation of many incomprehensive measures. However, for dynamic systems, error spectrum is difficult to analyze an estimator's performance since it is a three-dimension graphics for the whole time horizon. Accordingly, dynamic error spectrum was proposed to relieve error spectrum to dynamic systems. The key of dynamic error spectrum is combining error spectrum into one point at a time instant, which unfortunately suffers from information loss. To improve this, a new metric called enhanced dynamic error spectrum is proposed in this paper. Then, both the additive form and the multiplicative form of enhanced dynamic error spectrum are given under different application backgrounds. Finally, the concrete application of the metrics and qualities of the above form are illustrated by numerical examples. The simulation shows that more important information are considered by enhanced dynamic error spectrum and the proposed measure can also be applied easily to dynamic systems.

1. Introduction

In recent years, estimation performance evaluation (EPE) has received a great amount of attention due to their increasing use in estimation (see, e.g., [1–7,11,12]), fusion (see, e.g., [13,14]) and target tracking (see, e.g., [15–18]). In EPE, the root mean square error (RMSE) is one of the most popular incomprehensive performance measures since it is the most natural finite-sample approximation of the standard deviation. However, RMSE is easily dominated by large error terms. For instance, if all 100 terms of estimation error are around 1 except for one term of 1000, then the RMSE is approximate to 100. Clearly, the RMSE is unreasonable because it ignores small errors even if they are in an overwhelmingly large number [1,2]. So, the average Euclidean error (AEE) was suggested to replace the RMSE in many applications [1]. Although AEE has many advantages, it is still affected by extreme values (in the above example, AEE is approximate to 10). Therefore, many incomprehensive performance measures were elaborated in [1] such as the harmonic average error (HAE), geometric average error (GAE), median error and error mode. Furthermore, a robust metric called iterative mid-range error (IMRE) was given in [6] since these metrics are not robust.

Unfortunately, all of the above-listed metrics can reflect only one aspect of the estimator performance. Thus, three comprehensive performance measures—the error spectrum (ES), desirability level, and relative concentration and deviation measures were proposed in [2,4,5]. Among these metrics, the ES can reveal much information about the estimation performance because it is an aggregation of many incomprehensive metrics.

However, the ES has some limitations and drawbacks. On one hand, it is not easy to calculate without the error distribution, although [2] along with its further discussion [8] focused the computation of ES. Therefore, calculation is still a problem for the ES. To overcome this problem, two approximation algorithms was proposed in [9] based on the Gaussian mixture and power means error. On the other hand, the ES is presented in a form being suitable for parameter estimation directly; that is, for dynamic systems, it is hard to analyze the estimator's performance because the ES is a three-dimension (3D) graphics over the total time span.

For this reason, the dynamic error spectrum (DES) is presented in [10,11], which is combining the ES into one point at a time instant so as to transform the three dimensional graphics into two dimensional graphics. Besides, the DES has been used to analyze the performance of the IMM algorithm [11]. In fact, the DES is the average height of the ES. Obviously, the advantage is that the estimation performance can be visually reflected in the dynamic

* Corresponding author. Tel.: +86 18182425800.
E-mail addresses: peng_weishi@163.com (W.-S. Peng), ywfang2008@sohu.com (Y.-W. Fang), chaibaodong@126.com (D. Chai).

http://dx.doi.org/10.1016/j.ijleo.2016.01.105

system, meanwhile the disadvantage is that much important information will be lost when utilizing the DES. Furthermore, the DES provides a ruler only to measure how large the estimation error is. Recall that the least-squares (LS) estimation and minimum mean square error (MMSE) estimation differ from the maximum likelihood (ML) estimation and maximum a posteriori (MAP) estimation in their underlying ideas. The former seeks an estimator that has the smallest error, while the latter uses the "most frequently occurred" value of the estimatee as the estimator [1]. Although the ML and MAP estimators may have a larger average error, they may have a higher probability of being close to the estimatee. This has important implications while choosing an estimation method for a particular application. Take the estimation for interception/weapon control as an example, two estimators are considered and their average error equals to zero. Estimator 1 delivers most estimates within the kill zone (such that the target can be destroyed) but has some really bad misses; estimator 2 misses the kill zone quite often but has few really bad misses and thus has a zero average error. Obviously, it does the first estimator that should be chosen for this application due to the more concentration of the estimation errors. Moreover, the application of the DES to evaluate estimators for such applications as weapon control and interception is not so appropriate. Thus, a worthwhile problem is how to evaluate an estimator considering both the estimation error and probability in dynamic systems.

In this paper, the main contribution is that a new metric called enhanced dynamic error spectrum (EDES), is proposed to evaluate an estimator in dynamic systems. And also two forms of EDES are presented as well as their pros and cons, the first form is the additive form of enhanced dynamic error spectrum (A-EDES), which is a natural way if some prior knowledge about the estimator can be obtained; the second form is the multiplicative form of enhanced dynamic error spectrum (M-EDES), which is given without the prior knowledge. Finally, numerical examples are provided to illustrate the utility and effectiveness of the EDES metrics.

This paper is organized as follows. ES and DES are summarized in Section 2, the EDES and its two forms are presented in Section 3 to evaluate an estimator. Numerical examples are provided in Section 4 to illustrate the utility and effectiveness of the EDES metric. Section 5 concludes this paper.

2. Summary of error spectrum and dynamic error spectrum

2.1. Error spectrum

According to [2,23], let the (possibly vector-valued) estimation error $\tilde{\theta}$ of a (point) estimator $\hat{\theta}$ be $\tilde{\theta} = \theta - \hat{\theta}$ where θ is the estimand (i.e., the quantity to be estimated). We denote $e = ||\tilde{\theta}||$ or $e = ||\tilde{\theta}||/||\theta||$ as the absolute or relative estimation error norm, where $||\cdot||$ can be 1-norm or 2-norm. Then, for $r \in [-\infty, +\infty]$, at a time instant t, the ES is defined as

$$S(r,t) = [E(e(t)^r)]^{1/r} = \left[\int e(t)^r dF(e(t))\right]^{1/r} = \begin{cases} \left[\int e(t)^r f(e(t)) de\right]^{1/r} & \text{if } e(t) \text{ is continuous} \\ \left[\sum p_i e(t)_i^{\,r}\right]^{1/r} & \text{if } e(t) \text{ is discrete} \end{cases} \quad (1)$$

where $F(e(t))$, $f(e(t))$, and p_i are the cumulative distribution function (CDF), probability density function (PDF), and probability mass function (PMF), respectively.

From (1), for a fixed time t_0 it is clear that the ES includes several incomprehensive metrics as special cases when r is set to some specific values:

(a) $S(2, t_0) = (E[e(t_0)^2])^{1/2}$. Thus, for a discrete $e_i(t_0)$, $S(2, t_0)$ = RMSE.
(b) $S(0, t_0) \triangleq \lim_{r\to 0} S(r, t_0) = \exp(E[\ln \ e(t_0)])$. Thus, for a discrete $e_i(t_0)$, $S(0, t_0) \triangleq$ GAE.
(c) $S(1, t_0) = E[e(t_0)]$. Thus, for a discrete $e_i(t_0)$, $S(1, t_0)$ = AEE.
(d) $S(-1, t_0) = \frac{1}{E[1/e(t_0)]}$. Thus, for a discrete $e_i(t_0)$, $S(-1, t_0)$ = HAE.

In view of this, the notation r used in this paper is a real number that satisfies $r \in [-1, 2]$.

Certainly, ES is a curve for the state estimator of a dynamic system at any time instant. So it is a 3D figure over the total time span, which leads to the difficulty of EPE for dynamic system. Fortunately, DES has been proposed to solve this problem.

2.2. Dynamic error spectrum

In [10,11], if some prior knowledge about the weights $\{w_i\}_{i=1}^n$ corresponding to each given r_i, $r_i \in \{r_i\}_{i=1}^n$, can be obtained, where $\sum_{i=1}^n w_i = 1$, the weighted form of the DES, at a time t, can be simply summarized as

$$\text{DES}(w, r, t) = \sum_{i=1}^{n} S(r_i, t) w_i \quad (2)$$

Since the weights are difficult to obtain, another form is given by using average height of the ES, which is defined as

$$\text{DES}(r, t) = \frac{1}{r_n - r_1}\int_{r_1}^{r_n} S(r, t) dr \approx \frac{1}{n}\sum_{i=1}^{n} S(r_i, t) \quad (3)$$

Actually, the above form is dominated by large terms. Thus based on the balanced property of GAE, DES can be rewritten as

$$\text{DES}(r, t) = \exp\left(\frac{1}{r_n - r_1}\right)\int_{r_1}^{r_n} \ln \ S(r, t) dr \approx \exp\left[\frac{1}{n}\sum_{i=1}^{n} \ln S(r_i, t)\right] \quad (4)$$

Clearly, we can see that the DES is compressing the r-axis of the error spectrum curve to one point at a time instant. So, much information of the r-axis will be lost as using the DES. From another perspective, the key of this metric is to use the average height of the ES curve, as shown in Example 1.

Example 1. Assume that we have two estimators $\tilde{\theta}(t)_1$ and $\tilde{\theta}(t)_2$, at an interval of time $t \in [0, 30]$, their estimation errors are follows the two distributions, i.e.,

$$p(\tilde{\theta}(t)_1) = 0.5N(-0.8, 0.19) + 0.5N(0.8, 0.19)$$

$$p(\tilde{\theta}(t)_2) = 0.5N(0, 1.8) + 0.5N(0, 0.8)$$

where $N(\mu, \sigma^2)$ is a Gaussian distribution with mean μ and variance σ^2.

Then for each estimator, their ES over the time interval $t \in [0, 30]$, PDF curves, DES curves and ES curves (at a time instant t_0) are given in Figs. 1–4, respectively.

As shown in Figs. 1 and 3, it is hard to see which estimator performs better. Intuitively, substituting ES into the Eq. (3) yields

$$\text{DES}(r, t)_1 = \text{DES}(r, t)_2 = 0.720$$