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Quantum transmissivity of one-dimensional mirror structure photonic crystals

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ABSTRACT

In this paper, we have studied the transmission characteristic of one-dimensional mirror structure photonic crystals (MSPCs) $(AB)^N D_1 D_2 D_3 (BA)^N$ with the quantum theory method. We find there are some sharp peaks (quantum transmissivity $T = 1$) in the PBG of MSPCs, and the number of sharp peaks is added with the increasing of thickness, refractive index and numbers of defect layer, which is beneficial to design the optic filter of multiple channel.

1. Introduction

Photonic crystals (PCs) originate from (a) theoretical work of Yablonovitch, and (b) John's experimental work; both of which were published almost simultaneously in 1987 [1,2]. PCs are artificial dielectric or metallic structures in which the refractive index changes periodically in space. Such materials are unique in that their dielectric permittivity can change periodically in one, two or three dimensions at a spatial scale comparable with the light wavelength. This kind of periodic structure affects the propagation of electromagnetic waves in the similar way as the periodic potential in a semiconductor crystal affects the electron motion by defining allowed and forbidden electronic energy bands. Whether or not photons propagate through PC structures depends on their frequency. Frequencies that are allowed to travel are known as modes, and groups of allowed modes form bands. Disallowed bands of frequencies are called photonic band gaps (PBGs) [3,4]. The properties of PBGs in one-dimensional (1D) PCs have been proven to play an important role in some potential applications such as photonic devices, optical filters, resonance cavities, laser applications, high reflecting omnidirectional mirrors, and the optoelectronic circuits [5–10].

In Refs. [11,12], we have studied the quantum transmission property of 1D PCs with quantum theory method. In this paper, we have optimized the quantum theory method, and obtained the simplified quantum transfer matrix, but the calculation results were unchanged. We use the improved quantum theory method to research the MSPCs quantum transmissivity, and obtain some valuable results for MSPCs, which are beneficial to design the optic filter of multiple channel.

2. The quantum wave equation and probability current density of photon

The quantum wave equations of free and non-free photon have been obtained in Refs. [13,14], they are

$$i\hbar \frac{\partial}{\partial t}\vec{\psi}(\vec{r}, t) = c\hbar\nabla \times \vec{\psi}(\vec{r}, t), \tag{1}$$

and

$$i\hbar \frac{\partial}{\partial t}\vec{\psi}(\vec{r}, t) = c\hbar\nabla \times \vec{\psi}(\vec{r}, t) + V\vec{\psi}(\vec{r}, t), \tag{2}$$

where $\vec{\psi}(\vec{r}, t)$ is the vector wave function of photon, and V is the potential energy of photon in medium. In the medium of refractive index n, the photon's potential energy V is [13,14]

$$V = \hbar\omega(1 - n). \tag{3}$$

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The conjugate of Eq. (2) is

$$-i\hbar\frac{\partial}{\partial t}\vec{\psi}^*(\vec{r},t) = c\hbar\nabla\times\vec{\psi}^*(\vec{r},t) + V\vec{\psi}^*(\vec{r},t). \tag{4}$$

Multiplying the Eq. (2) by $\vec{\psi}^*$, the Eq. (4) by $\vec{\psi}$, and taking the difference, we get

$$i\hbar\frac{\partial}{\partial t}(\vec{\psi}^*\cdot\vec{\psi}) = c\hbar(\vec{\psi}^*\cdot\nabla\times\vec{\psi} - \vec{\psi}\cdot\nabla\times\vec{\psi}^*) = c\hbar\nabla\cdot(\vec{\psi}\times\vec{\psi}^*), \tag{5}$$

i.e.,

$$\frac{\partial\rho}{\partial t} + \nabla\cdot J = 0, \tag{6}$$

where

$$\rho = \vec{\psi}^*\cdot\vec{\psi}, \tag{7}$$

and

$$J = ic\vec{\psi}\times\vec{\psi}^*, \tag{8}$$

are the probability density and probability current density, respectively.

By the method of separation variable

$$\vec{\psi}(\vec{r},t) = \vec{\psi}(\vec{r})f(t) = \vec{\psi}(\vec{r})e^{-i\omega t}, \tag{9}$$

the time-dependent Eq. (2) becomes the time-independent equation

$$c\hbar\nabla\times\vec{\psi}(\vec{r}) + V\vec{\psi}(\vec{r}) = E\vec{\psi}(\vec{r}), \tag{10}$$

where E and V are the energy and potential energy of photon in medium, respectively.

3. The quantum transmissivity and reflectivity

In Refs. [11,12], we have taken curl in Eq. (10), and obtain second-order differential equation (13). In this paper, we should not taken curl in Eq. (10), and directly solve the Eq. (10) in one-dimensional Photonic crystals. Taking Eq. (3) into Eq. (10), we have

$$c\hbar\nabla\times\vec{\psi}(\vec{r}) = (E-V)\vec{\psi}(\vec{r}) = \hbar\omega n\vec{\psi}(\vec{r}), \tag{11}$$

i.e.,

$$\nabla\times\vec{\psi}(\vec{r}) = \frac{\omega}{c}n\vec{\psi}(\vec{r}), \tag{12}$$

and

$$\nabla\times\vec{\psi}(\vec{r}) = \frac{\omega}{c}\vec{\psi}(\vec{r}). \tag{13}$$

The Eqs. (12) and (13) are the quantum wave equations of photon in medium and vacuum, respectively. Eq. (12) can be written as

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \psi_x & \psi_y & \psi_z \end{vmatrix} = \frac{\omega}{c}n(\psi_x\vec{i} + \psi_y\vec{j} + \psi_z\vec{k}) \tag{14}$$

i.e.,

$$\begin{cases} \frac{\partial\psi_z}{\partial y} - \frac{\partial\psi_y}{\partial z} = \frac{\omega}{c}n\psi_x \\ \frac{\partial\psi_x}{\partial z} - \frac{\partial\psi_z}{\partial x} = \frac{\omega}{c}n\psi_y \\ \frac{\partial\psi_y}{\partial x} - \frac{\partial\psi_x}{\partial y} = \frac{\omega}{c}n\psi_z \end{cases}, \tag{15}$$

we consider the photon travels along with the x-axis in one-dimensional Photonic crystals, which is shown in Fig. 1. The wave vector $k_y = k_z = 0$ and $k_x \neq 0$. Since the photon wave is transverse wave, we have

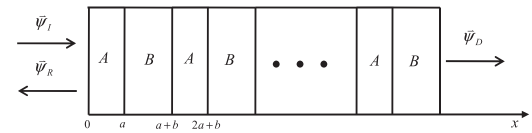

Fig. 1. The structure of one-dimensional photonic crystals.

$$\begin{cases} \psi_x = 0 \\ \psi_y = \psi_{0y}e^{ikx} \\ \psi_z = \psi_{0z}e^{ikx} \end{cases}, \tag{16}$$

where $k = (\omega/c)n$ is the wave vector of photon in medium. Substituting Eq. (16) into Eq. (15), we get

$$\begin{cases} -ik\psi_z = \frac{\omega}{c}n\psi_y = k\psi_y \\ ik\psi_y = \frac{\omega}{c}n\psi_z = k\psi_z \end{cases}, \tag{17}$$

i.e.,

$$\psi_y = -i\psi_z, \quad \psi_z = i\psi_y, \tag{18}$$

by Eq. (15), we get

$$\frac{d\psi_y}{\psi_y} = i\frac{\omega}{c}ndx \tag{19}$$

by integration, we obtain $\psi_y(x)$

$$\psi_y(x) = \psi_{0y}e^{i\frac{\omega}{c}nx} = \psi_{0y}e^{ikx} \tag{20}$$

and the $\psi_z(x)$ is

$$\psi_z(x) = \psi_{0z}e^{ikx} = i\psi_{0y}e^{ikx} \tag{21}$$

the total including time wave function of photon in medium is

$$\vec{\psi}(x,t) = \psi_y(x,t)\vec{j} + \psi_z(x,t)\vec{k} = \psi_{0y}e^{i(kx-\omega t)}\vec{j} + \psi_{0z}e^{i(kx-\omega t)}\vec{k}. \tag{22}$$

the total including time wave function of photon in vacuum is

$$\vec{\psi}(x,t) = \psi_y(x,t)\vec{j} + \psi_z(x,t)\vec{k} = \psi_{0y}e^{i(Kx-\omega t)}\vec{j} + \psi_{0z}e^{i(Kx-\omega t)}\vec{k}. \tag{23}$$

where $K = (\omega/c)$ is the wave vector of photon in vacuum.

For one-dimensional Photonic crystals, we should define and calculate its quantum transmissivity and quantum reflectivity. The one-dimensional PCs structure is shown in Fig. 1.

In Fig. 1, $\vec{\psi}_I$, $\vec{\psi}_R$, $\vec{\psi}_D$ are the wave functions of incident, reflection and transmission photon, respectively, they can be written as

$$\vec{\psi}_I = F_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j} + F_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{24}$$

$$\vec{\psi}_R = F'_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j} + F'_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{25}$$

$$\vec{\psi}_D = D_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j} + D_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{26}$$

where F_y, F_z, F'_y, F'_z, D_y, and D_z are incident, reflection and transmission amplitudes of y and z components. We obtain the solutions (22)–(26) are the same as the solutions (20)–(22) in Refs. [11,12].

By Eq. (22), the probability current density can be written as

$$J = ic\vec{\psi}\times\vec{\psi}^* = 2c|\psi_z|^2\vec{i} = 2c|\psi_{0z}|^2\vec{i}, \tag{27}$$

where

$$\psi_z = \psi_{0z}e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \tag{28}$$

the ψ_{0z} is ψ_z amplitude.

For the incident, reflection and transmission photon, their probability current density J_I, J_R, J_T are

$$J_I = 2c|F_z|^2, \tag{29}$$