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Dynamics of a low-order atmospheric circulation chaotic model

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ABSTRACT

In this paper, the dynamics of the Lorenz model of general circulation of the atmosphere is investigated. The global exponential attractive set and positively invariant set are obtained based on well-known Lyapunov stability theory and the function extremum theory for this low-order atmospheric circulation chaotic model. Furthermore, the exponential rate of the trajectories going from the exterior of the attractive set to the interior of the attractive set is also obtained. At last, numerical studies are provided to illustrate the effectiveness of the presented scheme.

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1. Introduction

Chaos and chaotic systems have been intensively studied in the last decades. Chaotic systems are dynamical systems described by nonlinear differential equations and they are strongly sensitive to initial conditions and the parameters. The well-known Lorenz system, Rössler system, Chua's circuit system, Chen system, Lü system and other chaotic dynamical systems are served as important chaotic models for the study of chaos [1–8].

Due to great potential applications in electrical engineering, information processing and so on, it is important to analyze the dynamical behaviors and properties of the new chaotic systems.

Ultimate boundedness is one of the fundamental concepts of dynamical systems, which plays an important role in investigating the uniqueness of equilibrium, the asymptotic stability, the existence of the periodic solution, estimating the Lyapunov dimension of chaotic attractors, estimating the Hausdorff dimension of the chaotic attractor, chaos control, chaos synchronization and so on. Ultimate boundedness is also very important for engineering applications [9,10], since it is very difficult to predict the existence of hidden attractors and they can lead to crashes. Due to the significance of scientific and engineering background of the famous Lorenz system, Leonov first studied the global boundedness of the Lorenz system and obtained many important results in [2,3]. Motivated by Leonov et al., the bounds of other chaotic systems, including the complex Lorenz chaotic system [11], the synchronous motor system [12], the Lü system [13], the brushless DC motor system [14], the monetary chaotic system [15] and the family of Lorenz-like systems [16] were also studied. Technically, this is also a very difficult task [11–16]. Furthermore, there is no unified method for construsting the Lyapunov functions to study the bounds of chaotic systems.

The novelty of this article is that not only do we get the ultimate bounds of the low-order atmospheric circulation chaotic model in [17–20], but we also get the trajectories of the system going from the exterior of the trapping set to the interior of the trapping set.

The rest of this study is organized as follows, the mathematical model is given in Section 2. In Section 3, we choose the proper Lyapunov function to prove the existence of positively invariant sets and global exponential attractive sets of the low-order atmospheric circulation model [17–20]. Some numerical simulations are also given in Section 3. Section 4 gives conclusions.

2. Mathematical model

The atmospheric component of the coupled low-order model is composed of the following three ordinary differential equations [17–20]:

$$\begin{cases} \dfrac{dx}{dt} = -y^2 - z^2 - ax + aF, & (1a) \\ \dfrac{dy}{dt} = xy - bxz - y + G, & (1b) \\ \dfrac{dz}{dt} = xz + bxy - z, & (1c) \end{cases} \qquad (1)$$

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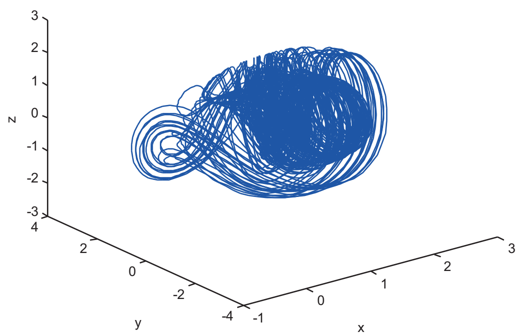

Fig. 1. Chaotic attractor of system (1) in the phase space.

where x is the zonal wind, y and z are the amplitudes of cosine and sine phases of large scale eddies, respectively. Interactions between the mean flow and eddies are represented by the amplification of eddies at the expense of the zonal flow intensity (first terms on the right-hand side of (1)), displacement of the eddies by the zonal flow (second terms in (1b) and (1c)), as well as mechanical damping (second term in (1a) and third terms in (1b) and (1c)). F and G terms represent diabatic heating contrasts between the low- and high-latitude ocean (F term) and seasonally varying zonal heating zonal difference between the land and ocean in the mid-latitudes (G term). Formulations for F and G are detailed later. And a, b, G, F are positive parameters of system (1). When $a = \left(1/4\right)$, $b = 4$, $G = 1$, $F = 8$, the system (1) can generate a chaotic attractor, as shown in Figs. 1 and 2 (also see [17–20]).

3. Main results

The local bifurcation of the equilibrium point, periodic solutions, homoclinic orbits, heteroclinic orbits, limit cycles, attractors and chaotic behaviors are studied in the papers [17–20]. But, the global exponential attractive sets of the low-order atmospheric circulation model (1) have not been studied yet.

In this section, we will discuss the positively invariant sets and global exponential attractive sets of system (1) for $\forall a > 0, b > 0, G > 0, F > 0$. We have the following theorem.

Theorem 1. There exists a positive number $L > 0$, such that the following set

$$\Pi = \left\{ (x, y, z) \middle| x^2 + y^2 + z^2 \le L \right\} \tag{2}$$

is the ultimate bound and positively invariant set of system (1).

Proof. Define the following Lyapunov function

$$V(x, y, z) = x^2 + y^2 + z^2, \tag{3}$$

And we can get

$$\begin{aligned} \left.\frac{dV(x,y,z)}{dt}\right|_{(1)} &= 2\left(x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}\right), \\ &= 2x\left(-y^2 - z^2 - ax + aF\right) + 2y\left(xy - bxz - y + G\right) \\ &\quad + 2z(xz + bxy - z), \\ &= -2ax^2 - 2y^2 - 2z^2 + 2aFx + 2Gy, \end{aligned}$$

Obviously, the surface Γ:

$$\Gamma = \left\{ (x, y, z) \middle| \frac{\left(x - \left(F/2\right)\right)^2}{\left(\left(aF^2 + G^2\right)/4a\right)} + \frac{\left(y - \left(G/2\right)\right)^2}{\left(\left(aF^2 + G^2\right)/4\right)} + \frac{z^2}{\left(\left(aF^2 + G^2\right)/4\right)} = 1 \right\}, \tag{4}$$

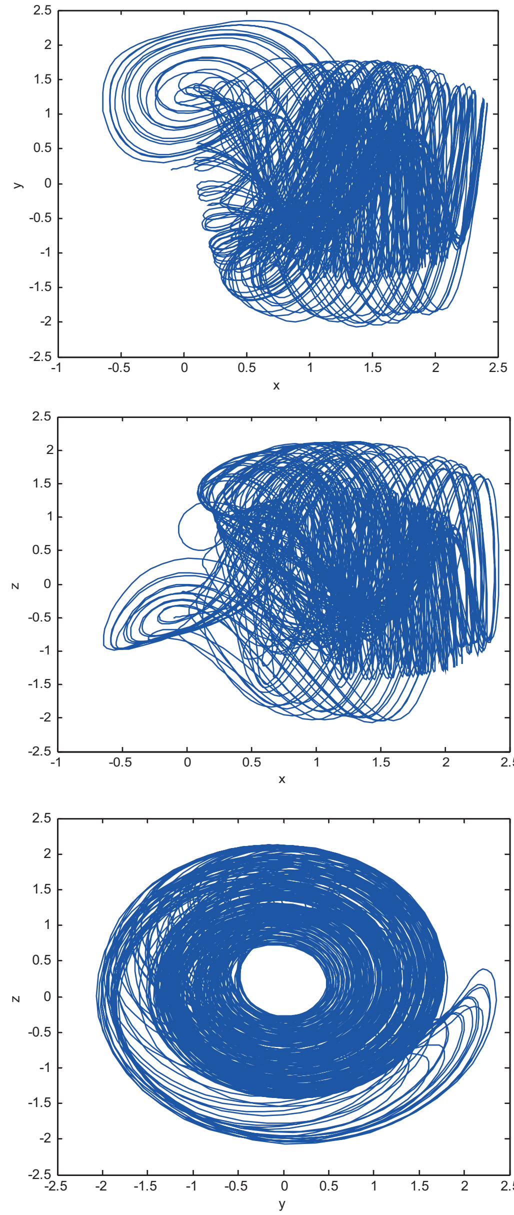

Fig. 2. Chaotic attractor of system (1) in x–y, x–z, and y–z planes.

is an ellipsoid in R^3 for $\forall a > 0$, $b > 0$, $G > 0$, $F > 0$. Since the $V(x, y, z)$ is a continuous function and Γ is a bounded closed set, then function (3) can reach its maximum value $\max\limits_{(x,y,z)\in\Gamma} V(x, y, z) = L$ on the surface Γ in (4). Obviously,

$$\left\{ (x, y, z) \middle| V(x, y, z) \le \max_{(x,y,z)\in\Gamma} V(x, y, z) = L, (x, y, z) \in \Gamma \right\}$$

contains solutions of system (1). It is obvious that (2) is the ultimate bound and positively invariant set for system (1).

This completes the proof.

Theorem 1 claims that the trajectories of system (1) are ultimate boundedness. But it does not gives the rate of the trajectories of system (1) going from the exterior of the trapping set to the interior

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