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Enhancing quantum entanglement by means of weak measurement under various environments

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ABSTRACT

We investigate the relation between dissipation and coherent effects in the dynamics of quantum entanglement (QE) between two qubits by means of weak measurement. Under weak measurement, the increase or decrease of QE closely relates with the dissipation. The amplitude of QE will only be elevated through enhancing the measurement strength in weak correlated or no correlated instantaneous modes, and the entanglement sudden death (ESD) is retarded effectively. In the correlated dephasing mode, however, the amplitude of QE will decrease due to the increases of measurement strength. The periods for entanglement death, as well as the dark and bright in the concurrence are not changed. Therefore, the environmental relevance helps to enhance the amplitude of QE and delay the entanglement death.

1. Introduction

One of the most remarkable features of quantum physics is quantum entanglement (QE). In recent years, theoretical and experimental progresses have indicated that QE is not only the most important primary resource for quantum information, such as quantum teleportation [1,2], quantum dense coding [3], but also the cornerstone to realize quantum communication and quantum computation. In comparison with the classical information science of the classical physics, quantum information science based on QE has many advantages. It can not only greatly enhance the speed and capacity of information processing, but also resolve the information processing functions which might not be achieved in the past. Therefore, it becomes one of the key technologies to implement the quantum information.

On the other hand, the inevitable QE between quantum system and environment is the main reason for quantum decoherence and quantum information loss. The phenomenon, that leads the quantum systems to fully get rid of QE in a limited time due to the effect of environmental dissipation, is known as entanglement sudden death (ESD) [4]. In order to ensure the correctness of quantum logic operation and quantum computation, people have done a lot of researches about avoiding ESD and proposed new plans to achieve QE in some cases. But many problems about disentanglement remain to be solved.

Quantum measurement is essential for the process of quantum information processing. Traditional quantum measurement technologies exploit von Neumann measurement to gain information. Undoubtedly, they may not be the optimal measurement scheme since these methods cause unrecoverable destructions to the quantum states as the information is acquired. In Ref. [5], a brand-new weak measurement was constructed. Although the gained information was less than the von Neumann measurement, the perturbation was much lesser. More significantly, the quantum states after weak measurement were recoverable. Pryde et al. managed to measure the polarization weak value of photons in linear optical system [6]. Korotkov and Jordan proposed a scheme of weak measurement in superconducting qubits [7], and experimentally realized by Katz [8]. Recently, it is pointed out that weak measurements, together with quantum measurement reversals, protect quantum states of singlet qubit systems [9,10] and two-qubit systems [11,12]. Very recently, weak measurement and a subsequent reverse operation are found to be able to improve the fidelity of teleportation when one or both qubits of a maximally entangled state shared between the sender and the receiver undergo amplitude damping [13]. The protocol is shown to work even for the Werner state.

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In this paper, we implement weak measurement on coupling qubits. Our purpose is to compare and investigate the dynamical behavior of QE in an open system. Our paper is organized as follows. In Section 2, a description of the suggested model is introduced. Subsequently, in Section 3, we develop the theory to study the dynamics of QE of two interacting qubits and calculate the time evolution of QE under the influence of environmental perturbations by means of weak measurement. Finally, we conclude in Section 4.

2. The model

Considering the coupled qubits (denoted as A and B) that initially in entangled state, the Hamiltonian is given by ($\hbar = 1$) [14]

$$H_0 = \omega_0 \left(\sigma_A^z + \sigma_B^z\right) + g\left(\sigma_A^+ \sigma_B^- + \sigma_B^+ \sigma_A^-\right), \quad (1)$$

where σ_j^z, $\sigma_j^\pm$ ($j = A, B$) are Pauli matrices for the qubits, g represents interacting strength between two qubits.

The qubits are inevitably affected by uncontrollable degrees of freedom from environment. In general, the time evolution of quantum open system can be described by a super-operator L. Taking into account the coupling between system and external environment, the master equation describing the dynamical evolution of system writes [15]

$$\dot{\rho} = -i\left[H_0, \rho\right] + L\rho, \quad (2)$$

where the first term is a usual unitary evolution, and the second one describes all possible transitions that the open system may undergo due to the interaction with its reservoir. The super-operator L depends on the environments of the two qubit system. In this paper, we mainly discuss the instantaneous decay mode, correlated dephasing decay mode and correlated instantaneous decay mode, which will be described by different super-operators. The specific expressions of super-operators could be seen in the fourth part.

The solution of Eq. (2) depends on the initial state. In this paper, we consider the initially entangled qubits to be in a mixed state given by the density matrix

$$\rho(0) = \frac{1}{3}\begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \mu & \delta & 0 \\ 0 & \delta^* & \nu & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix}, \quad (3)$$

where α, β, μ, ν and δ are the initial parameters of qubits with the relations $\alpha + \beta + \mu + \nu = 3$. For convenience, assume $\mu = \nu = |\delta| = 1$. Here $\delta = e^{i\chi}\sqrt{\mu\nu}$ is the coherence of single photon, where the initial phase χ determines the QE part of the state.

To minimize the influence on the original quantum state, a measurement is introduced that induces a partial collapse of the quantum state, which is the so-called weak measurement. Our purpose is to suppress decoherence and obtain more QE. So, we adopt the null-result weak measurement, which is different from amplitude damping, to analyze the effect on the evolution of entangled qubits. The map of weak measurement on qubit j can be represented as [16]

$$\begin{cases} |0\rangle_j\langle 0| \to |0\rangle_j\langle 0|, \quad |1\rangle_j\langle 1| \to (1-x_j)|1\rangle_j\langle 1| \\ |0\rangle_j\langle 1| \to \sqrt{1-x_j}|0\rangle_j\langle 1|, \quad |1\rangle_j\langle 0| \to \sqrt{1-x_j}|1\rangle_j\langle 0| \end{cases} \quad (4)$$

where $0 \le x_j \le 1$ is the weak measurement strength, which is proportional to the time the environment is monitored. Practically, weak measurement on a qubit can be done via monitoring its environment by an ideal detector.

For simplicity, we assume $x_A = x_B = x$. So the initial quantum state after weak measurement can be rewritten as

$$\rho^{wk}(0) = \frac{A(x)}{3}\begin{pmatrix} \alpha(1-x)^2 & 0 & 0 & 0 \\ 0 & (1-x) & (1-x)e^{i\chi} & 0 \\ 0 & (1-x)e^{-i\chi} & (1-x) & 0 \\ 0 & 0 & 0 & 1-\alpha \end{pmatrix}, \quad (5)$$

with normalization coefficient $A(x)$

$$A(x) = \frac{3}{3 - 2(1+\alpha)x + \alpha x^2} \quad (6)$$

3. Decay of two-qubit entanglement

In order to describe the dynamic evolution of QE, we adopt the concurrence defined by Wootters to quantify the degree of QE for any bi-partite system. It is well-known that any reliable measure of QE will yield the same result. The concurrence $C(t)$ is defined as [17]

$$C(t) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (7)$$

where λ_j are the eigenvalues of $W = \rho(t)\tilde{\rho}(t)$ in no increasing order by magnitude with $\tilde{\rho}(t) = \left(\sigma_y^{(1)} \otimes \sigma_y^{(2)}\right)\rho^*(t)\left(\sigma_y^{(1)} \otimes \sigma_y^{(2)}\right)$. In this paper, the concurrence is given by

$$C^{\mathrm{wk}}(t) = 2\max\left\{0, \tilde{C}^{\mathrm{wk}}(t)\right\} \quad (8)$$

where

$$\tilde{C}^{\mathrm{wk}}(t) = 2\left(\left|\rho_{23}^{\mathrm{wk}}(t)\right| - \sqrt{\rho_{11}^{\mathrm{wk}}(t)\rho_{44}^{\mathrm{wk}}(t)}\right) \quad (9)$$

3.1. Effects of weak measurement in instantaneous decay mode

The instantaneous decay of quantum system is particularly important to quantum engineering, which results in loss of coherence in the system and therefore it is a fundamental constraint on quantum correlation. In this section, we consider an instantaneous decay environmental model and investigate the effect of weak measurement on dynamics of QE between two qubits. For the instantaneous decay mode, $L\rho$ is expressed as

$$L\rho = -\sum_{j=A,B}\frac{\gamma_j}{2}\left(\sigma_j^+\sigma_j^-\rho - 2\sigma_j^-\rho\sigma_j^+ + \rho\sigma_j^+\sigma_j^-\right), \quad (10)$$

where γ_A, γ_B is the instantaneous decay rate from the environment of qubit A and B, respectively.

For the sake of discussion, we assume $\gamma_A = \gamma_B = \gamma$ in this section. In an instantaneous decay environmental mode, the solution of (3) in the initial state (5) is direct. Using the dynamical evolutions of the density matrix elements, we can obtain

$$\tilde{C}_1^{\mathrm{wk}}(t) = \frac{2A(x)}{3}e^{-\gamma t} \times \left\{(1-x)\sqrt{\cos^2\chi + \sin^2\chi\cos^2(2gt)} - \sqrt{(1-\alpha)f_1(t)}\right\}, \quad (11)$$

with

$$f_1(t) = x^2 - 2\alpha x + \alpha + 2(1-x)(1-e^{-\gamma t}) + (1-\alpha)(1-e^{-\gamma t})^2. \quad (12)$$

One can see that $\tilde{C}_1^{\mathrm{wk}}(t)$ not only depends on the instantaneous decay rate γ but also relates with the interaction g between qubits, the initial phase χ, and the measurement strength x.

As shown in Fig. 1, the phenomenon of dark and bright of entanglement happens for $g \neq 0$ in the dynamical evolution of the