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Compound-combination synchronization of five chaotic systems via nonlinear control

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ABSTRACT

Based on combination synchronization and compound synchronization, a novel kind of compound-combination synchronization scheme among five chaotic systems has been investigated in this paper. On the basic of the adaptive control and the stability theory, the mixed system which is a compound system among three drive chaotic systems can realize the synchronization with the novel system, which is a combination system between two response chaotic systems. The corresponding theoretical analysis and numerical simulations are provided to prove the validity and feasibility of the proposed control scheme. The unpredictability and complexity of the compound system of three drive systems can additionally strengthen the security of secure communication. The transmitted signals can be split into several different parts loaded in three drive systems to improve the reliability of secure communication.

1. Introduction

Chaos synchronization behavior has been received more and more attractions, which is of great significance in theory and practical value. The researchers have made great efforts due to its potential application in many scientific and engineering fields in the last decades. Kinds of synchronization phenomenons for these chaotic systems have been studied and a lot of meaningful results have been gained to our future application, such as complete synchronization [1], anti-synchronization [2], generalized synchronization [3–7], phase synchronization [8], anti-phase synchronization [9], lag synchronization [10], partial synchronization [11], projective synchronization [12–17], time scale synchronization [18], etc.

A scheme has been proposed to achieve combination synchronization among three different chaotic systems, where the combination system of two drive systems is realized the synchronization with one response system [19–22]. Based on the combination scheme, the researchers utilize the general benefits of the combination synchronization scheme, and have extended it to combination–combination synchronization between a combination system of two drive systems and a combination system of two response systems [23–25]. To enhance the security of communication, a novel kind of compound synchronization among four chaotic systems is investigated, where the mixed system of three drive systems can realize the synchronization with one response system, three drive systems have been conceptually divided into two categories: scaling drive systems and base drive systems [26–28]. However, there are the existing literature works focused on combination synchronization, combination–combination synchronization and compound synchronization. To the best of our knowledge, there are few studies on compound-combination synchronization of five chaotic systems. The study on compound-combination synchronization of five chaotic systems is an opportunity and challenge in the field of chaotic synchronization.

Inspired by the above discussions, we employ the general benefits of combination synchronization and compound synchronization schemes, and extend them to compound-combination synchronization of five chaotic systems. In the paper, a novel kind of compound-combination synchronization of five chaotic systems is investigated by combining the adaptive control and the stability theory of nonlinear systems. Compound-combination synchronization shows that the compound system of three drive systems can synchronize with the novel system, which is a combination system of two response systems. In general, the level of security will greatly dependent on the complexity level of the drive dynamical system and the formation of the driving signals as well as the modulation scheme used. The unpredictability

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and complexity of the compound system of three drive systems can additionally strengthen the security of communication. The transmitted signals can be split into several different parts loaded in three drive systems to improve the reliability of secure communication.

Compared with prior work, the paper extends combination synchronization of three chaotic systems [19–22], combination–combination synchronization of four chaotic systems [23–25] and compound synchronization schemes of four chaotic systems [26–28] to compound-combination synchronization of five chaotic systems. In contrast to combination synchronization in [19–22], if the scaling system is a constant, and there is a response system, then compound-combination synchronization of five chaotic systems will be changed into combination synchronization of three chaotic systems. This is a significant improvement upon the existing combination synchronization in [19–22]. Compared with combination–combination synchronization in [23–25], if the scaling system is a constant, then compound-combination synchronization of five chaotic systems will be changed into combination–combination synchronization of four chaotic systems. Combination–combination synchronization is included in compound-combination synchronization of five chaotic systems as a special case. This is a meaningful extension on the proposed combination–combination synchronization in [23–25]. Different from compound-combination synchronization scheme, if there is a response system, then compound-combination synchronization of five chaotic systems will be changed into compound synchronization of four chaotic systems [26–28]. Compound-combination synchronization problem cannot be viewed as a simple collective form of five chaotic systems. This is because, unlike the above synchronization scheme of multi chaotic systems, the synchronization is only considered as a single case. Three drive systems and two response systems are combined to realize compound-combination synchronization, which can need a more delicate and complex scheme.

2. Problem formulation

In the section, we firstly design the scheme of compound-combination synchronization with three drive systems and two response systems. The first drive system is given as follows:

$$\dot{x}_1 = f_1(x_1). \tag{1}$$

The second drive system is written as

$$\dot{x}_2 = f_2(x_2). \tag{2}$$

The third drive system is given as

$$\dot{x}_3 = f_3(x_3), \tag{3}$$

and the first response system is described by

$$\dot{y}_1 = g_1(y_1) + u. \tag{4}$$

The second response system is written by

$$\dot{y}_2 = g_2(y_2) + u^*, \tag{5}$$

where $x_1=(x_{11},x_{12},\ldots,x_{1n})^T$, $x_2=(x_{21},x_{22},\ldots,x_{2n})^T$, $x_3=(x_{31},x_{32},\ldots,x_{3n})^T$, $y_1=(y_{11},y_{12},\ldots,y_{1n})^T$, and $y_2=(y_{21},y_{22},\ldots,y_{2n})^T$ are five state vectors of systems (1), (2), (3), (4) and (5), respectively; f_1, f_2, f_3, g_1, g_2: $R^n \to R^n$ are five continuous vector functions; $u=(u_1,u_2,\ldots,u_n)^T$ and $u^*=(u_1^*,u_2^*,\ldots,u_n^*)^T$: $R^n \times R^n \times \ldots \times R^n \to R^n$ are two controllers of two response systems (4) and (5) which will be designed, respectively.

Definition 1. For three drive systems (1)–(3) and two response systems (4), (5), it is said that compound-combination synchronization can be achieved, if there exist the scaling constants a_i, b_i, c_i, k_i and l_i, such that

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty} [a_i x_{1i}(b_i x_{2i} + c_i x_{3i}) - k_i y_{1i} - l_i y_{2i}] = 0, \tag{6}$$

e_i is the compound-combination synchronization error ($i=1,2,\ldots,n$), the drive system (1) is referred as the scaling drive system, two drive systems (2) and (3) are referred as the base drive systems, two systems (4) and (5) are the response systems.

Remark 1. Assume the scaling system is a constant a, and there is only one response system, in other words, $x_{1i}=a$, $y_{1i}=0$ or $y_{2i}=0$, the synchronization error is $\lim_{t\to\infty} e_i = \lim_{t\to\infty} [a_i a(b_i x_{2i} + c_i x_{3i}) - k_i y_{1i}] = 0$ or $\lim_{t\to\infty} e_i = \lim_{t\to\infty} [a_i a(b_i x_{2i} + c_i x_{3i}) - l_i y_{2i}] = 0$, then the compound-combination synchronization problem of five chaotic systems will be changed into the combination synchronization problem of three chaotic systems [19–22].

Remark 2. Assume the scaling system is a constant a, in other words, $x_{1i}=a$, the synchronization error is $\lim_{t\to\infty} e_i = \lim_{t\to\infty} [a_i a(b_i x_{2i} + c_i x_{3i}) - k_i y_{1i} - l_i y_{2i}] = 0$, then the compound-combination synchronization problem of five chaotic systems will be changed into the combination–combination synchronization problem of four chaotic systems [23–25].

Remark 3. Assume there is only one response system, in other words, $y_{1i}=0$ or $y_{2i}=0$, the synchronization error is $\lim_{t\to\infty} e_i = \lim_{t\to\infty} [a_i x_{1i}(b_i x_{2i} + c_i x_{3i}) - k_i y_{1i}] = 0$ or $\lim_{t\to\infty} e_i = \lim_{t\to\infty} [a_i x_{1i}(b_i x_{2i} + c_i x_{3i}) - l_i y_{2i}] = 0$, then the compound-combination synchronization problem of five chaotic systems will be changed into the compound synchronization problem of four chaotic systems [26–28].

Remark 4. Combination synchronization, combination–combination synchronization and compound synchronization are included in compound-combination synchronization of five chaotic systems as a special case. The unpredictability and complexity of the compound system of three drive systems can additionally strengthen the security of secure communication. The transmitted signals can be split into several different parts loaded in three drive systems to improve the reliability of secure communication.