Optik 126 (2015) 503–506

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Various projective synchronization phenomena in two different variable time-delayed systems related to optical bistable devices

Cun-Fang Feng[a,b,*], Jian-Yong Guo[a,b], Yan-Rong Tan[a,b]

[a] School of Electronic and Electrical Engineering, Wuhan Textile University, Wuhan, Hubei 430073, China
[b] Research Center of Nonlinear Science, Wuhan Textile University, Wuhan, Hubei 430073, China

ARTICLE INFO

Article history:
Received 22 January 2014
Accepted 16 January 2015

Keywords:
Projective-anticipatory synchronization
Projective-lag synchronization
Variable time delays
Time-delayed systems

ABSTRACT

We analytically and numerically analyze the existence of various projective (projective-anticipatory, projective and projective-lag) synchronization between two different modulated time-delayed chaotic systems in this paper. The innovation of our work is that we report on projective-anticipatory and projective-lag synchronization of two different variable time-delayed chaotic systems, while other existing work only mentioned projective synchronization in such systems. Based on the theory of Krasovskii–Lyapunov stability, we derive the sufficient stability condition through theoretical analysis. Mackey–Glass and Ikeda models related to optical bistable or hybrid optical bistable devices are taken for examples to verify the validity and effectiveness of the proposed method.

1. Introduction

Synchronization, a typical collective behavior in nature, means that two or more systems share a common dynamical behavior. Synchronization in chaotic dynamics systems has obtained much attention particularly due to their potential application in secure communication, modelling brain activity, system identification and so on [1–4]. Now the notion of synchronization is extended far beyond complete synchronization, such as generalized synchronization [5], phase synchronization [6], anti-synchronization [7], projective synchronization [8], projective-anticipatory and projective-lag synchronization [9–11], and so on. Projective synchronization implies that the corresponding state variables of the master-slave systems evolve in a proportional scale, $y(t)=\alpha x(t)$; Projective-anticipatory synchronization appears as a coincidence of shifted-in-time states of two systems and their amplitudes are correlated by the scaling factor, $y(t)=\alpha x(t+\tau)$, $\tau>0$; For projective-lag synchronization, in contrast to projective-anticipatory synchronization, the slave system lags the master, $y(t)=\alpha x(t-\tau)\equiv x_\tau(t)$ with a positive τ.

Projective-anticipatory, projective and projective-lag synchronization has aroused great interests in time-delayed chaotic systems with constant delays [12–17]. However recently it was reported that time-delayed systems with constant delays are not secure because constant delays can be exposed by autocorrelation, average mutual information and filling factor [18–20], et al. Thus projective-anticipatory, projective and projective-lag synchronization in variable time-delayed systems has attracted attentions [21–27]. These works mainly focus on the projective synchronization of two identical systems. However, in practice, it is hardly the case that the structure of master and slave chaotic systems can be assumed to be identical. Therefore, investigation of the problem of projective-anticipatory, projective and projective-lag synchronization between two different chaotic systems with variable time delays is an important research topic. However, there are few theoretical results about projective-anticipatory, projective and projective-lag synchronization between two different time-delayed chaotic systems with modulated time delays, since they consist of different dynamical structures and the existing of modulated delays. To the best our knowledge, until now, only projective synchronization between two different variable time-delayed systems was observed in Refs. [28,29], where projective-anticipatory and projective-lag synchronization were not involved.

Motivated by the above discussions, the purpose of the present work is to investigate the projective-anticipatory, projective and projective-lag synchronization of two different variable time-delayed systems. The main contribution of this paper lies that we observe projective-anticipatory and projective-lag synchronization between two different variable time-delayed chaotic systems. The rest of the paper is organized as follows. Section 2 describe a general method for projective-anticipatory, projective and projective-lag synchronization between two different modulated time-delay systems with the proper controller; Stability analysis using Krasovskii–Lyapunov theory is also presented in this

* Corresponding author. Tel.: +86 15926360851.
E-mail address: cunffeng@aliyun.com (C.-F. Feng).

http://dx.doi.org/10.1016/j.ijleo.2015.01.009
0030-4026/

section. In Section 3 some representative examples are introduced to verify the effectiveness of the proposed scheme. Finally, conclusions and discussions are made in the last section.

2. Theoretical analysis

We consider the following first-order nonlinear equation related to optical bistable or hybrid optical bistable devices with a variable time delay $\tau_1(t)$:

$$\dot{x}(t) = -ax(t) + b_1 f(x_{\tau_1(t)}), \quad (1)$$

where, $x \in R^n$ is the state variable, $\tau_1(t)$ is the delay time, a and b_1 are system parameters, and $f: R^n \to R^n$ is a nonlinear function.

Eq. (1) is considered as the master system. The slave system with a controller $u \in R^n$ is given by

$$\dot{y}(t) = -ay(t) + b_2 g\left(\frac{y_{\tau_2(t)}}{\alpha}\right) + u. \quad (2)$$

In this paper, we consider two time delays $\tau_1(t)$ and $\tau_2(t)$ as a function of time, instead of constant delays, as

$$\tau_1(t) = \tau_{10} + r_1 \sin(\omega_1 t),$$

$$\tau_2(t) = \tau_{20} + r_2 \sin(\omega_2 t), \quad (3)$$

where $\tau_{1,2}(t) = \tau_{10,20} + r_{1,2}\sin(\omega_{1,2}t)$ are the variable delay times, $\tau_{10,20}$ are the zero-frequency components, $r_{1,2}$ are the amplitudes and $\omega_{1,2}/2\pi$ are the two frequency of the modulation, respectively.

One find that the systems (1) and (2) can be synchronized on the synchronization manifold $y = \alpha x_{\tau_d}$, under the appropriate controller u.

Definition. For the systems (1) and (2), it is said that they are synchronized in the sense of (a) projective-anticipatory synchronization as $\tau_d < 0$; (b) projective synchronization as $\tau_d = 0$; (c) projective-lag synchronization as $\tau_d > 0$, respectively, if there exists a nonzero constant scaling factor α (a proportional relation) such that

$$\lim_{t\to\infty} \|e\| = \lim_{t\to\infty} \|y - \alpha x_{\tau_d}\| \to 0, \quad (4)$$

where $\|\cdots\|$ is the Euclidian norm, e represents the synchronization error.

In the following, the main goal is to design the active controller u in Eq. (2) to stabilize the error variables at the origin. Due to $e = y - \alpha x_{\tau_d}$, thus the error dynamical system between the master system (1) and slave system (2) is

$$\dot{e} = \dot{y} - \alpha\dot{x}_{\tau_d} = -ay + b_2 g\left(\frac{y_{\tau_2(t)}}{\alpha}\right) + u - \alpha[-ax_{\tau_d} + b_1 f(x_{\tau_1+\tau_d})]. \quad (5)$$

In order to observe projective-anticipatory, projective and projective-lag synchronization between systems (1) and (2), we design the controller u as follows

$$u = \alpha b_1 f(x_{\tau_1+\tau_d}) - b_2 g(x_{\tau_2+\tau_d}). \quad (6)$$

Combining Eq. (6), the error dynamics (5) becomes

$$\dot{e} = -ae + b_2 g\left(\frac{y_{\tau_2}}{\alpha}\right) - b_2 g(x_{\tau_2+\tau_d}); \quad (7)$$

due to $e = y - \alpha x_{\tau_d}$, we obtain $y_{\tau_2}/\alpha = x_{\tau_2+\tau_d} + e_{\tau_2}/\alpha$, therefore the error system (7) becomes

$$\dot{e} = -ae + \frac{b_2}{\alpha} \cdot g'(x_{\tau_2+\tau_d}) \cdot e_{\tau_2}. \quad (8)$$

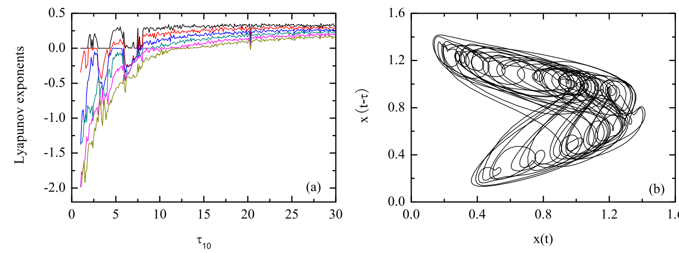

Fig. 1. (a) First six largest Lyapunov exponents of the system (13) versus the parameter τ_{10}. (b) Phase space diagram as $\tau_{10} = 11$, after letting transients relax.

To examine the stability of synchronization manifold $y = \alpha x_{\tau_d}$, we use the Krasovskii–Lyapunov stability theory. Consider a positive definite Lyapunov functional of the form

$$V(t) = \frac{1}{2}e^2(t) + \frac{\sigma}{2(1-\epsilon)} \int_{-\tau_2(t)}^{0} e^2(t+\theta)d\theta, \quad (9)$$

where σ is a positive definite constant such that $\sigma \geq (1-\epsilon)/(1-\delta)$ and $0 < \epsilon < 1$. $\tau'_2(t)$ is bounded by $\delta \neq 1$. The time derivative of (9) is

$$\dot{V}(t) \leq e\dot{e} + \frac{\sigma}{2(1-\epsilon)}[e^2 - (1-\delta)e^2_{\tau_2(t)}]. \quad (10)$$

Substituting Eq. (8) into (10), we obtain

$$\begin{aligned} \dot{V}(t) &\leq e[-ae + \frac{b_2}{\alpha} \cdot g'(x_{\tau_2+\tau_d}) \cdot e_{\tau_2}] + \frac{\sigma}{2(1-\epsilon)}[e^2 - (1-\delta)e^2_{\tau_2(t)}] \\ &\leq -e^2\left[\frac{\sigma}{2(1-\epsilon)}(1-\delta)X^2 - \frac{b_2}{\alpha}g'(x_{\tau_2+\tau_d})X + a - \frac{\sigma}{2(1-\epsilon)}\right], \end{aligned} \quad (11)$$

where $X = e_{\tau_2}/e$. According to [30], the sufficient condition for the solution $e = 0$ of the variable time-delayed system $\dot{e}(t) = -r(t)e + s(t)e_{\tau(t)}$ is $r(t) > s^2(t)/2 + 1/2(1-\delta)$ and $s^2(t)/2 + 1/2(1-\delta) < -r(t)$. For the error dynamical system (8), the sufficient condition for synchronization is

$$a > \frac{b_2^2(\sup(g'(x_{\tau_2+\tau_d})))^2}{2\alpha^2} + \frac{1}{2(1-\delta)}$$

and

$$-a > \frac{b_2^2(\sup(g'(x_{\tau_2+\tau_d})))^2}{2\alpha^2} + \frac{1}{2(1-\delta)}. \quad (12)$$

Under this condition, by changing the sign of τ_d, we can observe projective-anticipatory, projective and projective-lag synchronization, respectively.

3. Application of the above mentioned scheme

In this section, Mackey–Glass model and Ikeda model will be taken for examples to verify the above mentioned scheme.

The dynamical equation of the Mackey–Glass system is given by [31]

$$\dot{x}(t) = -ax(t) + \frac{b_1 x_{\tau_1(t)}}{1 + x^{10}_{\tau_1(t)}}, \quad (13)$$

where $\tau_1(t)$ is the delay parameter and a, b_1 are the usual parameters of the Mackey–Glass system. For $a = 1$, $b_1 = 3$, $r_1 = 0.1$, $\omega_1 = 0.002$, the first six largest Lyapunov exponents versus the parameter τ_{10} is shown in Fig. 1(a). We can see that the system (13) displays hyperchaotic at $\tau_{10} = 11$. Fig. 1(b) shows phase space diagram corresponding to system (13) with $\tau_{10} = 11$.