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Conditions for the concurrent refraction of the ordinary and extraordinary rays along the optical axis in absorbing uniaxial media

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ABSTRACT

When a plane wave propagating in an isotropic medium is refracted in the interface with a uniaxial transparent medium in such a way that the ordinary ray propagates along the optical axis, then the extraordinary ray necessarily propagates along the optical axis too. In this situation the medium behaves as an isotropic medium with refractive index n_ω. In general this is not the case with absorbing uniaxial media unless some conditions are also fulfilled. In this work we obtain the necessary conditions that must be accomplished in order to have both the refracted ordinary and extraordinary rays propagating along the optical axis so that the uniaxial absorbing medium behaves like an isotropic one.

1. Introduction

Light propagation in anisotropic media has been extensively studied, partly motivated by the interest in the fundamental, physical behavior of these materials but also because of the sustained increase in practical applications both in transparent [1–10] and absorbing [11–20] media.

In anisotropic media the displacement vector $\boldsymbol{D}$ and the electric field $\boldsymbol{E}$ are related by $\boldsymbol{D} = \varepsilon\,\boldsymbol{E}$ where ε is the dielectric tensor. There always exists a coordinate system *XYZ*, called system of principal axes, in which this tensor is diagonal. In the system of principal axes we can define the principal refractive indices $n_{x,y,z}$ so that $n_{x,y,z} = \sqrt{\varepsilon_{x,y,z}/\varepsilon_o}$. In case of uniaxial media the optical axis is a symmetry axis that coincides with one of the principal axis and defines a privileged direction. In fact, if a wave propagates along this particular direction, the medium behaves as if it were isotropic. If the principal axis Z is the optical axis then the principal refractive indices are $n_x = n_y = n_\omega$ and $n_z = n_\varepsilon$. Here n_ω and n_ε are called ordinary and extraordinary indices respectively.

Optical anisotropy is of particular interest in case of light refraction in the boundary between isotropic and anisotropic media. In general, a plane wave propagating in an isotropic medium and falling on to the interface with a uniaxial medium is refracted into two waves with different directions of propagation and different refractive indices. One of the two indices depends on the medium but not on the direction of propagation of the wave. This index is called ordinary index and the wave that propagates with that index ordinary wave. The other refractive index, the extraordinary index, depends both on the medium and on the direction of propagation of the wave. The wave that propagates with this index is called extraordinary wave. If the plane of incidence contains the optical axis then there is a range of possible orientations of the optical axis for which it is possible to find an angle of incidence so that the ordinary and extraordinary refracted waves propagate in the direction of the optical axis and with the same refractive index n_ω.

We are interested in studying if a similar behavior can be observed in case of uniaxial absorbing media. Taking into account absorption considerable increases the mathematical complexity of the physical description. Regarding absorbing media, the usual approach is to use Fresnel's equation and its solutions but substituting the real refractive indices by their complex counterparts $\tilde{n}_{1,2} = n_{1,2} - ik_{1,2} = n_{1,2}(1 - i\chi_{1,2})$ with n the (real) refractive index, k the (real) absorption coefficient and χ the absorption index. Since refracted waves in absorbing media are in general inhomogeneous we will represent the direction of propagation with a complex vector $\tilde{\boldsymbol{s}}$ with $\tilde{\boldsymbol{s}} = \boldsymbol{S}_R + i\boldsymbol{S}_I$. Using the Fresnel's complex equation and requiring the complex refractive index of the extraordinary wave to be equal to the refractive index of the ordinary wave $\tilde{n}_\omega$, it is obtained, in contrast to the case of transparent media, that only in case of normal incidence and if the optical axis is perpendicular to the interface we can have the refracted ordinary and extraordinary waves propagating in the direction of the optical axis. Clearly, taking the limit of vanishingly small absorption will not recover the solution obtained in transparent uniaxial media.

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The apparent discontinuity between both solutions disappears if we take into account that a wave is refracted along the optical axis when the direction of phase propagation coincides with that direction.

In general it is found that in absorbing media when the direction of phase propagation of the refracted ordinary wave is the direction of the optical axis then the refracted extraordinary wave propagates with different direction of propagation of the phase and different refractive index. However, under some circumstances, the extraordinary wave tends to be refracted in the direction of the optical axis, much as in the transparent case. This happens in the limit of vanishingly small absorptions, i.e. as the medium becomes transparent. It also occurs when the direction of the incident wave approaches to the normal to the interface and finally, it also happens when the medium is isotropic in both refractive indices and absorption coefficients. There are also cases in which the two refracted ordinary and extraordinary waves can propagate in the (real) direction defined by the optical axis even for non-transparent anisotropic media in conditions of non-normal incidence. In this case, although both waves propagate in the same direction, their refractive indices are not the same [21].

According to the behavior exhibited by the direction of propagation of the phase of the refracted waves [21] we may also expect a different behavior respect to the direction of propagation of the rays along the direction of the optical axis in uniaxial absorbing media respect to what is observed in case of uniaxial transparent media.

In case of transparent media the direction of propagation of the ray and the direction of propagation of the phase of the ordinary wave always coincide. Therefore if the ordinary refracted wave propagates in the direction of the optical axis then the ray will also propagate in this direction. Respect to the extraordinary wave, if the optical axis is not contained in the plane of incidence, the ray it will not be in general contained in this plane [10]. Even in the cases in which the plane of incidence contains the optical axis the direction of the Poynting vector, although contained in that plane, does not coincide in general with the direction of propagation of the phase. If we require the ordinary refracted wave to have the direction of the optical axis, then the optical axis must be contained in the plane of incidence. Then if ϕ is the angle that the direction of propagation of the phase of the extraordinary wave makes with the optical axis, then the angle that the direction of the Poynting vector makes with the optical axis ϕ_R is [1]:

$$tg\phi_R = \frac{n_\omega^2}{n_\epsilon^2} tg\phi \tag{1}$$

This direction coincides with the direction of the optical axis when the direction of propagation of the phase also coincides with the optical axis ($\phi = 0$).

In summary, in case of an interface between an isotropic transparent medium and an uniaxial transparent medium there exists a particular direction for the incident wave for which, respect to refraction, the uniaxial medium behaves as an isotropic medium with index n_ω.

In this work we will study the direction of the Poynting vector of the refracted waves in uniaxial absorbing media. We will obtain the precise conditions that guarantee that both the ordinary and extraordinary rays propagate in the same direction so that the direction of the optical axis, like in the case of transparent media, acts as a direction of isotropy.

2. Poynting vectors in uniaxial absorbing media

In absorbing media the refracted waves are in general inhomogeneous. If we represent by $\tilde{\boldsymbol{F}}$ any of the field vectors (displacement

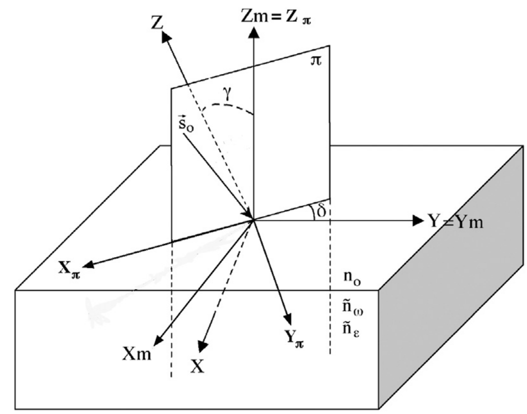

Fig. 1. A plane wave falls with an angle of incidence α_o from an isotropic medium with index n_o to a uniaxial absorbing medium with indices $\tilde{n}_\omega$ and $\tilde{n}_\varepsilon$; $X_mY_mZ_m$ is the coordinate system of the medium, XYZ the coordinate system of principal axes and $X_\pi Y_\pi Z_\pi$ is defined so that $Y_\pi Z_\pi$ is the plane of incidence π.

vector, electric field or magnetic field) then it can be written as [14]:

$$\begin{aligned}\tilde{\boldsymbol{F}} &= F\tilde{\boldsymbol{u}}_F \exp\left[i\omega\left(t - \frac{\tilde{n}}{c}\boldsymbol{r}\cdot\tilde{\boldsymbol{s}}\right)\right]\exp(i\theta_F)\\ &= F\tilde{\boldsymbol{u}}_F \exp\left[-\frac{\omega}{c}n\boldsymbol{r}\cdot(\chi\boldsymbol{S}_R - \boldsymbol{S}_I)\right] exp\left[i\omega\left(t - \frac{n}{c}\boldsymbol{r}\cdot(\boldsymbol{S}_R - \chi\boldsymbol{S}_I)\right)\right]\\ &\quad exp(i\theta_F)\end{aligned} \tag{2}$$

where the real constant F is the field amplitude, $\tilde{\boldsymbol{s}}(\tilde{\boldsymbol{s}} = \boldsymbol{S}_R + \boldsymbol{S}_I)$ is the unitary complex vector that represents the direction of propagation, the complex unitary vector $\tilde{\boldsymbol{u}}_F = \boldsymbol{u}_{RF} + i\boldsymbol{u}_{IF}$ along the complex direction of $\tilde{\boldsymbol{F}}$ gives the polarization of the field, θ_F is the initial phase, $(\chi\,\boldsymbol{S}_R - \boldsymbol{S}_I)$ a vector along the direction of maximum attenuation and $(\boldsymbol{S}_R + \chi\boldsymbol{S}_I)$ a vector along the direction of propagation of the phase.

In general neither the ordinary ray nor the extraordinary rays are contained in the plane of incidence [13]. Even in the simpler case of isotropic media it may happen that the direction of the ray is not contained in that plane depending on the polarization of the incident field. Only if the displacement vector is contained in the plane of incidence or it is linearly polarized then the Poynting vector will be necessarily contained in the plane of incidence [14] and its direction will coincide with the direction of propagation of the phase.

We consider the interface between an isotropic medium with refractive index n_o and a uniaxial absorbing medium with principal refractive indices $\tilde{n}_\omega$ and $\tilde{n}_\varepsilon$. We define the coordinate system X_m, Y_m and Z_m adapted to the interface so that the X_m and Y_m axes lay on the interface and the Z_m axis is perpendicular to it. This coordinate system will be referred hereafter as the coordinate system of the medium. Respect to the system of principal axes, since the optical axis Z is a symmetry axis of the uniaxial medium, the X and Y axes can be any pair of mutually orthogonal axes contained in the plane perpendicular to the optical axis Z [15]. This fact allows us to simplify the calculations by choosing the Y axis coincident with the Y_m as shown in Fig. 1. With this arrangement the X_mZ_m and XZ planes are the same plane. Let us further suppose that the optical axis Z makes an angle γ with Z_m. Let π be the plane of incidence, determined by the X_π and Z_π axes and δ the angle that it makes with the Y_mZ_m plane.

Let us consider an incident wave that falls from an isotropic medium with refractive n_o to a uniaxial absorbing medium so that the direction of propagation of the phase of the refracted ordinary wave is along the optical axis. For this to happen the optical axis Z has to be in the plane of incidence π and the angle γ that the