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A two-step indirect FDTD method for calculating anisotropic ferrite

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ABSTRACT

In this paper, the author presented a two-step indirect FDTD method which can be used for processing magnetically anisotropic media, and then derived the iterative formula. According to the introduction of intermediate variables, this paper is divided into two parts by the specific form of frequency-domain permeability, and then the complex frequency-dependent entries can be replaced by new variables. First, the author derived the relationship of intermediate variables in the frequency domain, which is then transformed to the time domain, further the author deduced iteration formula of magnetic fields, so as to solve the complex magnetic constitutive relations. By this, we solved the electromagnetic field problems in ferrite successfully. As a verification, using the method the reflection coefficient and transmission coefficient of an infinite ferrite slab were calculated, the numerical results show that the method is simple and practical, which is easy to implement and saves computing time.

1. Introduction

In recent years, the study of magnetized ferrite is being paid more and more attention [1,2]. Ferrite is a kind of dispersive medium material, it manifests anisotropy when external magnetic field exist. As dispersive media, the permeability of ferrite will change with the incident wave frequency, and will be high in high-frequency band. And ferrite's electromagnetic properties varies with size and direction of the field. The unique characteristics of the ferrite make it widely used in the design of microwave integrated circuits and devices. It is difficult to solve the permeability of magnetized ferrite strictly for its anisotropy appeared, then it would be important that analysis of electromagnetic problems with numerical methods such as finite-difference time-domain method (FDTD). Currently the main FDTD methods for dispersive media are recursive convolution (RC) method [3], Z transform (ZT) [4], the shift operator (SO) method [5,6], the current density convolution (JEC) [7], auxiliary differential equation (ADE) method [8,9], Runge–Kutta exponential time differencing method (RKETD) [10,11], exponential time differencing (ETD) [12,13] method and so on. It is common that these methods are more used in research of electrical dispersive medium such as plasma than the study of magnetic dispersive medium, and not to mention the study of magnetic anisotropic medium. For the study of the magnetic anisotropy medium, it needs to promote the methods above. In 1998, Chen et al. calculated circulation features of ferrous targets in free space using FDTD method [14]. In 2007, Yang and others provided an analysis of anisotropic ferrite ball's backscattering and other issues with Pade-FDTD and (RC-FDTD) [15,16]. In 2009, Wang et al. analyzed metal object covered ferrite to back radar cross section (RCS) by using SO-FDTD [17,18]. The reference [16] needs to calculate the time domain convolution and calculation process is complex, while the reference [17] requires the dispersive model for the rational fraction form, which limits its application in a certain range. This paper presents a two-step indirect FDTD method for magnetic anisotropy medium, which decomposed the constitutive relation of magnetic into two parts, then with an intermediate variable instead of the complex frequency-dependent entry, to solve electromagnetic problems about magnetic anisotropy by solving.

2. Presentation of new calculating method

Located along the z-axis direction of the external magnetic field, then the permeability of anisotropic ferrite can be expressed as the following tensor form [19,20]:

$$\mu_r(\omega) = \begin{bmatrix} \mu_{xx} & \mu_{xy} & 0 \\ \mu_{yx} & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} = \mu_0 \begin{bmatrix} \mu_r & j\mu_{rg} & 0 \\ -j\mu_{rg} & \mu_r & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} \tag{1}$$

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where,

$$\left.\begin{aligned} \mu_r &= 1 + \frac{T\omega_m}{T^2 - \omega^2} \\ \mu_{rg} &= \frac{\omega\omega_m}{T^2 - \omega^2} \\ \mu_{rz} &= 1 \end{aligned}\right\}, \quad T = \omega_0 + j\omega\alpha \tag{2}$$

Consider the propagation of uniform plane wave along the z direction, in the one-dimensional case, then $E_z = 0$, $B_z = 0$, $H_z = 0$. The partial derivatives of the field along the direction of x, y is 0. The Maxwell equations can be decomposed into component form as shown in the formulas (3-a) and (3-b):

$$\left.\begin{aligned} \frac{\partial E_x}{\partial t} &= -\frac{1}{\varepsilon_0}\frac{\partial H_y}{\partial z} \\ \frac{\partial B_y}{\partial t} &= -\frac{\partial E_x}{\partial z} \end{aligned}\right\} \tag{3-a}$$

$$\left.\begin{aligned} \frac{\partial E_y}{\partial t} &= \frac{1}{\varepsilon_0}\frac{\partial H_x}{\partial z} \\ \frac{\partial B_x}{\partial t} &= \frac{\partial E_y}{\partial z} \end{aligned}\right\} \tag{3-b}$$

The constitutive equations for ferrite:

$$\begin{bmatrix} B_x(\omega) \\ B_y(\omega) \end{bmatrix} = \begin{bmatrix} \mu_{xx}H_x(\omega) + \mu_{xy}H_y(\omega) \\ \mu_{yx}H_x(\omega) + \mu_{yy}H_y(\omega) \end{bmatrix} \tag{4}$$

For ease of programming, normalization equations is used in the derivation of FDTD formulas [21], and therefore the formulas (3) and (4) will no longer appear ε_0 and μ_0 in form. The following derivation is based on the normalized Eq. (2) into a normalized (4), we obtain:

$$\begin{bmatrix} H_x(\omega) \\ H_y(\omega) \end{bmatrix} = \begin{bmatrix} B_x(\omega) \\ B_y(\omega) \end{bmatrix} - \begin{bmatrix} J_x(\omega) \\ J_y(\omega) \end{bmatrix} \tag{5-a}$$

Its form in time domain:

$$\begin{bmatrix} H_x^n \\ H_y^n \end{bmatrix} = \begin{bmatrix} B_x^n \\ B_y^n \end{bmatrix} - \begin{bmatrix} J_x^n \\ J_y^n \end{bmatrix} \tag{5-b}$$

where,

$$\left.\begin{aligned} J_x(\omega) &= \frac{T\omega_m H_x + j\omega\omega_m H_y}{T^2 - \omega^2} \\ J_y(\omega) &= \frac{T\omega_m H_y + j\omega\omega_m H_x}{T^2 - \omega^2} \end{aligned}\right\} \tag{6}$$

Here T is same with Eq. (2); we can get:

$$\begin{aligned} [\omega_0^2 + 2\alpha\omega_0(j\omega) + (\alpha^2+1)(j\omega)^2] \cdot J_x(\omega) &= \omega_0\omega_m H_x(\omega) \\ &+ \alpha\omega_m(j\omega)H_x(\omega) + \omega_m(j\omega)H_y(\omega) \end{aligned} \tag{7}$$

$$\begin{aligned} [\omega_0^2 + 2\alpha\omega_0(j\omega) + (\alpha^2+1)(j\omega)^2] \cdot J_y(\omega) &= \omega_0\omega_m H_y(\omega) \\ &+ \alpha\omega_m(j\omega)H_y(\omega) + \omega_m(j\omega)H_x(\omega) \end{aligned} \tag{8}$$

The derivation procedure of $J_x(\omega)$ and $J_y(\omega)$ is as follows, change the Eqs. (7) and (8) into the time domain and finishing:

$$\begin{aligned} &\left(\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2}\right)J_x^n + \left(\omega_0 - \frac{2(\alpha^2+1)}{\Delta t^2}\right)J_x^{n-1} \\ &+ \left(\frac{\alpha^2+1}{\Delta t^2} - \frac{\alpha\omega_0}{\Delta t}\right)J_x^{n-2} = \left(\frac{\alpha\omega_m}{2\Delta t}\right)H_x^n + (\omega_0\omega_m)H_x^{n-1} \\ &+ \left(-\frac{\alpha\omega_m}{2\Delta t}\right)H_x^{n-2} + \left(\frac{\omega_m}{2\Delta t}\right)H_y^n + \left(-\frac{\omega_m}{2\Delta t}\right)H_y^{n-2} \end{aligned} \tag{9}$$

$$\begin{aligned} &\left(\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2}\right)J_y^n + \left(\omega_0 - \frac{2(\alpha^2+1)}{\Delta t^2}\right)J_y^{n-1} \\ &+ \left(\frac{\alpha^2+1}{\Delta t^2} - \frac{\alpha\omega_0}{\Delta t}\right)J_y^{n-2} = \left(\frac{\alpha\omega_m}{2\Delta t}\right)H_y^n + (\omega_0\omega_m)H_y^{n-1} \\ &+ \left(-\frac{\alpha\omega_m}{2\Delta t}\right)H_y^{n-2} + \left(-\frac{\omega_m}{2\Delta t}\right)H_x^n + \left(\frac{\omega_m}{2\Delta t}\right)H_x^{n-2} \end{aligned} \tag{10}$$

From (9) and (10) we can see that it need to use the current H_y^n and H_x^n to solve $J_x(\omega)$ and $J_y(\omega)$ due to the mutual coupling of the magnetic field makes it impossible to obtain solutions directly from Eqs. (9) and (10). Therefore substituted them into (5), and J_y^n got from Eq. (10) is adopted in the Eq. (9) then, we can get:

$$\begin{aligned} J_x^n = X_0 \cdot (&X_1 B_x^n + X_2 B_y^n + X_3 H_x^{n-1} + X_4 H_x^{n-2} + X_5 H_y^{n-1} + X_6 H_y^{n-2} \\ &+ X_7 J_x^{n-1} + X_8 J_x^{n-2} + X_9 J_y^{n-1} + X_{10} J_y^{n-2}) \end{aligned} \tag{11}$$

and,

$$\begin{aligned} J_y^n = Y_0 \cdot (&Y_1 B_y^n + Y_2 B_x^n + Y_3 H_y^{n-1} + Y_4 H_y^{n-2} + Y_5 H_x^{n-1} + Y_6 H_x^{n-2} \\ &+ Y_7 J_y^{n-1} + Y_8 J_y^{n-2} + Y_9 J_x^{n-1} + Y_{10} J_x^{n-2}) \end{aligned} \tag{12}$$

where the coefficients in Eqs. (11) and (12) are as follows:

$$X_0 = \frac{1}{1 + \left(\frac{\omega_m}{2\Delta t}\right)^2} \tag{13}$$

$$X_1 = \frac{\frac{\alpha\omega_m}{2\Delta t}}{\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}} + \frac{\left(\frac{\omega_m}{2\Delta t}\right)^2}{\left(\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}\right)^2} \tag{14}$$

$$X_2 = \frac{\frac{\omega_m}{2\Delta t}}{\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}} - \frac{\alpha\left(\frac{\omega_m}{2\Delta t}\right)^2}{\left(\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}\right)^2} \tag{15}$$

$$X_3 = \frac{\omega_0\omega_m}{\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}} \tag{16}$$

$$X_4 = -\frac{\frac{\alpha\omega_m}{2\Delta t}}{\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}} - \frac{\left(\frac{\omega_m}{2\Delta t}\right)^2}{\left(\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}\right)^2} \tag{17}$$

$$X_5 = -\frac{\frac{\omega_0\omega_m^2}{2\Delta t}}{\left(\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}\right)^2} \tag{18}$$

$$X_6 = -\frac{\frac{\omega_m}{2\Delta t}}{\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}} + \frac{\alpha\left(\frac{\omega_m}{2\Delta t}\right)^2}{\left(\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}\right)^2} \tag{19}$$

$$X_7 = -\frac{\omega_0^2 - \frac{2(\alpha^2+1)}{\Delta t^2}}{\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}} \tag{20}$$

$$X_8 = \frac{\frac{\alpha\omega_0}{\Delta t} - \frac{\alpha^2+1}{\Delta t^2}}{\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}} \tag{21}$$

$$X_9 = -\frac{\frac{\omega_m}{2\Delta t}\left[\omega_0^2 - \frac{2(\alpha^2+1)}{\Delta t^2}\right]}{\left(\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}\right)^2} \tag{22}$$

$$X_{10} = \frac{\frac{\omega_m}{2\Delta t}\left[\frac{\alpha^2+1}{\Delta t^2} - \frac{\alpha\omega_0}{\Delta t}\right]}{\left(\frac{\alpha\omega_0}{\Delta t} + \frac{\alpha^2+1}{\Delta t^2} + \frac{\alpha\omega_m}{2\Delta t}\right)^2} \tag{23}$$