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Time-frequency conversion based on linear time lens for measuring arbitrary waveform☆

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ABSTRACT

We propose a new technique to realize an optical time-frequency conversion system for ultrafast temporal processing that is based on linear time lens. The demonstrated time lens produces more than 25,000 of phase shift. Time-frequency conversion systems are realized by the combination of temporal quadratic phase modulation and group-velocity dispersion. We simulate the results with a time resolution of 27 fs over a time window of 1 ns, representing a large enough time-bandwidth product.

1. Introduction

Space-time duality is based on the analogy between the equations that describe the paraxial diffraction of light beams in space and the first-order temporal dispersion of optical pulses in a dielectric [1–3]. The duality can also be extended to consider imaging lenses: the use of quadratic phase modulation on a temporal waveform is analogous to the action of a spatial lens on the transverse profile of a spatial beam [4,5]. Pulse compression [6], timing-jitter reduction [7], time magnification [8], time cloaking [9] have been demonstrated using this concept. The effects of diffraction and spatial lenses on a beam of light are equivalent to the effects of dispersion and time lenses on a pulse of light. The time-frequency conversion process is best understood by noting the analogy between a temporal optical system manipulating pulses of light and a spatial optical system manipulating beams of light. Here we investigate a new regime in the interaction between optical pulses and time lenses. The optical pulse to be measured travels through one focal time of dispersion, supplied by the standard single-mode fiber, then is phase modulated by the time lens with a quadratic time phase shift. We know from Fourier optics that the field distributions at the front focal plane and output plane of the spatial lens are related by a Fourier transform, and we will show below that the same relation holds for a time lens. Also the output field distribution and its spectral frequency distribution are related by a Fourier transform [10,11].

We also provide a simulation experimental demonstration of the phenomenon using a linear time lens. Among other potential applications, spectro-temporal imaging can be applied for the measurement of the intensity temporal profile of ultrashort optical pulses using a conventional spectrum analyzer [12,13]. In contrast with other approaches, this method provides a fast, direct (single-shot), and unambiguous measurement of the temporal waveform, which is important in some relevant engineering applications such as modern high-speed communications. In this letter, we derive the conditions for achieving time-frequency conversion using a single time lens as well as the expressions governing this operation.

2. Principle of arbitrary waveform measurement

The fundamental principle behind temporal imaging is the analogy between the electric field propagation behavior in the cases of paraxial diffraction and narrow-band dispersion. In paraxial diffraction, the spatial envelope profile $E(x, y, z)$ of a monochromatic beam, $E(x, y, z, t) = E(x, y, z)\exp(i(\omega_0 t - kz))$, propagates according to [2]

$$\frac{\partial E}{\partial z} = -\frac{i}{2k}\left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2}\right). \tag{1}$$

This is easily solved in the transverse spatial frequency domain where the spectrum, $\varepsilon(k_x, k_y, z) = \mathcal{F}\{E(x, y, z)\}$, acquires a quadratic spectral phase upon propagation

$$\varepsilon(k_x, k_y, z) = \varepsilon(k_x, k_y, 0)\exp\left(\frac{iz(k_x^2 + k_y^2)}{2k}\right). \tag{2}$$

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For the narrow-band dispersion problem, we consider the evolution of a pulse envelope $A(z_0)$ for the plane wave profile, $A(z, t) = A(z_0)\exp(i(\omega_0 t - \beta z))$. The spectrum $\varepsilon(z, \omega)$ consists of the baseband spectrum $\mathcal{A}(z, \omega)$ of the envelope convolved up to the carrier ω_0, i.e., $\varepsilon(z, \omega) = \mathcal{A}(z, \Omega)\exp(-i\beta z)$, where $\Omega = \omega - \omega_0$. About the carrier at frequency ω_0, each spectral component represents a plane wave with the propagation constant $\beta(\omega)$, which can be expanded to second order in a Taylor series [8]

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta' + (\omega - \omega_0)^2 \frac{\beta''}{2}, \tag{3}$$

Where

$$\beta_0 = \beta(\omega_0), \quad \beta' = \left.\frac{d\beta(\omega)}{d\omega}\right|_{\omega=\omega_0} = \frac{1}{v_g(\omega_0)} \quad \text{and}$$

$$\beta'' = \left.\frac{d^2\beta(\omega)}{d\omega^2}\right|_{\omega=\omega_0} = \frac{d}{d\omega}\left.\left(\frac{1}{v_g(\omega)}\right)\right|_{\omega=\omega_0} \tag{4}$$

Here ω_0 is used to represent a general carrier frequency. In a parametric temporal imaging system, the carrier will change due to frequency conversion in the time lens and subsequent dispersive delay lines must be evaluated at the new carrier frequency.

By transforming to a traveling-wave coordinate system

$$\tau = (t - t_0) - \left(\frac{z - z_0}{v_g(\omega_0)}\right) \quad \text{and} \quad \xi = z - z_0, \tag{5}$$

where t_0 and z_0 are references in real time-space, defining the center of the waveform at $\tau = 0$ in the traveling-wave system, and $v_g(\omega_0)$ is the group velocity at the carrier frequency ω_0, the governing equation for the evolution of the envelope $A(\xi, \tau)$ is of the same form as that for paraxial diffraction (1)

$$\frac{\partial A}{\partial \xi} = \frac{i\beta''}{2}\left(\frac{\partial^2 A}{\partial \tau^2}\right), \tag{6}$$

Again, easily solved in the frequency domain, the spectrum of the pulse envelope $\mathcal{A}(\xi, \Omega) = \mathcal{F}\{A(\xi, \tau)\}$ is described independent of the carrier frequency and evolves during propagation according to [4]

$$A(\xi, \Omega) = A(0, \Omega)\exp\left(-\frac{i\phi''}{2}\Omega^2\right), \tag{7}$$

where, $\phi'' = \xi\beta''$, Ω is a baseband Fourier spectral component.

Any mechanism that produces a phase modulation that is quadratic phase shift in time can be considered as a time-domain analog to a space lens. Kolner showed that the ideal time lens has a quadratic phase versus time [3]:

$$H(\tau) = \exp\left(-i\frac{\tau^2}{2\phi''_f}\right). \tag{8}$$

Fig. 1 shows the concept of time lens with parabolic phase curve and linear chirp curve. An ideal lens imparts a quadratic phase $\varphi_f(\tau)$ to the signal such that $\varphi_f(\tau) = -\tau^2/2\phi''_f$, where ϕ''_f is the focal group-delay dispersion (GDD) associated with the time lens and is equal to the inverse of the second derivation of the phase.

The definitions of the input dispersion and time lens in a time-frequency conversion system are given in Table 1.

The time-frequency conversion system operation is determined by following the input field $E_0(0, \tau)$, an optical carrier at frequency ω_0 modulated by the pulse envelope $u_0(0, \tau)$,

$$E_0(0, \tau) = u_0(0, \tau)e^{i\omega_0\tau} \tag{9}$$

Firstly, the effect of the dispersion through a standard single-mode fiber with the propagation constant $\beta(\omega)$ distorts the input pulse envelope $u_0(0, \tau)$ into $u_1(\xi_1, \tau')$:

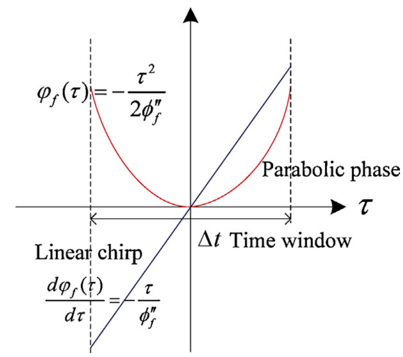

Fig. 1. The temporal profile of the phase and its derivative for ideal time lens.

Table 1
Equation describing the elements of a time-frequency conversion system.

	Time	Frequency
Input dispersion	$G_1(\xi_1, \tau) = \frac{1}{\sqrt{2\pi i\phi''_1}}\exp\left(i\frac{\tau^2}{2\phi''_1}\right)$	$\mathcal{G}_1(\xi_1, \Omega) = \exp\left(-i\frac{\phi''_1\Omega^2}{2}\right)$
Time lens	$H(\tau) = \exp\left(-i\frac{\tau^2}{2\phi''_f}\right)$	$\mathcal{H}(\Omega) = \sqrt{-2\pi i\phi''_f}\exp\left(i\frac{\phi''_f\Omega^2}{2}\right)$

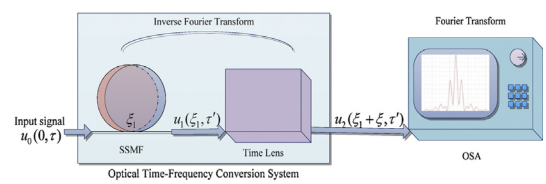

Fig. 2. Diagram of the proposed configurations for measuring optical pulses based on linear time Lens. SSMF = standard single-mode fiber, OSA = optic spectrum analyzer.

$$u_1(\xi_1, \tau') = \int_{-\infty}^{\infty} u_0(0, \tau)G_1(\xi_1, \tau - \tau')d\tau$$

$$= \frac{1}{\sqrt{2\pi i\phi''_1}}\int_{-\infty}^{\infty} u_0(0, \tau)\exp\left(i\frac{(\tau - \tau')^2}{2\phi''_1}\right)d\tau, \tag{10}$$

$$G_1(\xi_1, \tau - \tau') = \frac{1}{\sqrt{2\pi i\phi''_1}}\exp\left(i\frac{(\tau - \tau')^2}{2\phi''_1}\right). \tag{11}$$

The propagation constant $\beta(\omega)$ of the dispersive medium can be approximated by a three term Taylor series Eq. (3).

A sketch of the experimental setup for a time lens measurement is shown in Fig. 2. The pulse $u_1(\xi_1, \tau')$ is then quadratic phase modulated by the time lens. Eq. (8) showed that the ideal time lens has a quadratic phase versus time. After passing through the dispersive medium and the optical quadratic phase modulator, the output pulse $u_2(\xi_1 + \xi, \tau')$ is related to the input pulse $u_0(0, \tau)$ by

$$u_2(\xi_1 + \xi, \tau') = u_1(\xi_1, \tau')H(\tau')$$

$$= \frac{1}{\sqrt{2\pi i\phi''_1}}\exp\left(-i\frac{\tau'^2}{2\phi''_f}\right)\int_{-\infty}^{\infty} u_0(0, \tau)\exp\left(i\frac{(\tau' - \tau)^2}{2\phi''_1}\right)d\tau$$

$$= \frac{1}{\sqrt{2\pi i\phi''_1}}\exp\left(i\frac{\tau'^2}{2}\left(\frac{1}{\phi''_1} - \frac{1}{\phi''_f}\right)\right)$$

$$\int_{-\infty}^{\infty} u_0(0, \tau)\exp\left(i\frac{\tau^2}{2\phi''_1}\right)\exp\left(-i\frac{\tau\tau'}{\phi''_1}\right)d\tau. \tag{12}$$