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New approach for analysing the master equation with a damped harmonic oscillator and squeezed bath☆

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ABSTRACT

We solve the master equation with a damped harmonic oscillator and squeezed bath to obtain density operators' infinite operator–sum representation via a new approach, i.e., by virtue of the new thermo-entangled state representation that has a fictitious mode as a counterpart mode of the system mode. The corresponding Kraus operators from the point of view of quantum channel are also derived.

1. Introduction

A superoperator is a linear operator acting on a space of linear operators. Sometimes the term refers more specially to a completely positive map which does not increase or preserves the trace of its argument. This specialized meaning is used extensively in the field of quantum computing, especially quantum programming, as they characterise mappings between density matrices. For an open quantum system interacting with the environment, one uses superoperators to describe the evolution of density operator from an initial ρ_0 (pure state or mixed state) into a final state ρ. A superoperator plays a role of linear map from $\rho_0 \to \rho$, which has an operator–sum representation [1–3]

$$\rho = \sum_{n=0}^{\infty} M_n \rho_0 M_n^\dagger, \tag{1}$$

where the operators M_n and $M_n^\dagger$ are usually hermite conjugate to each other, and obey the normalization condition [4],

$$\sum_{n=0}^{\infty} M_n^\dagger M_n = 1. \tag{2}$$

M_n is named Kraus operator [5–8]. Such an operator–sum representation is the core of POVM (positive operator-valued measure).

In our very recent papers [9–12], based on the thermo entangled state representation, we have derived some density operators that are in infinite dimensional operator–sum forms, for example, those for describing amplitude-damping channel and laser process, the corresponding Kraus operators are also obtained and their normalization proved, which implies trace-preserving.

It is well-known that the standard theory of density operator equation (master equation) for a damped harmonic oscillator with squeezed bath is [13,14]

$$\begin{aligned}\frac{d\rho}{dt} &= -\frac{\lambda}{2}(N+1)(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a) \\ &\quad - \frac{\lambda}{2}N(aa^\dagger\rho - 2a^\dagger\rho a + \rho a a^\dagger) + \frac{\lambda}{2}M(a^2\rho - 2a\rho a + \rho a^2) \\ &\quad + \frac{\lambda}{2}M^*(a^{\dagger 2}\rho - 2a^\dagger\rho a^\dagger + \rho a^{\dagger 2})\end{aligned} \tag{3}$$

where λ denotes the decay rates of the single modes of the initial in the correlated environment. a and $a^\dagger$ are annihilation and creation operators,respectively. N denotes the average of thermal photons, and $M = |M|e^{-i\theta}$ characterizes the one-photon correlation property of the Gaussian correlated environment. In Eq. (3) positivity of the density matrix imposes the equality $|M|^2 = N(N+1)$.

Usually, this operator master equation is converted into an equivalent c-number equation by virtue of P-representation in the coherent state basis [15], which takes a considerable amount of work. Then an important and interesting question challenges us: is there a new approach for finding the infinite operator–sum

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representation of density operator in the Eq. (3) as $\rho=\sum_{n=0}^{\infty}M_n\rho_0M_n^{\dagger}$? The answer is affirmative.

In this paper, by using the new thermo entangled state representation we solve the master equation for a dissipative cavity with Kerr medium to obtain its density operator, and find its infinite operator–sum representation really possesses such a "strange structure", this may bring attention of theoreticians who might wonder if the general theory of superoperators should be modified.

The passage is arranged as follows: in Section 2, we reduced the master equation in Eq. (3) by introducing some complex parameters. In Sections 3 and 4, we introduce a new two-mode entangled state to solve this master equation. Our conclusions are given in the last section.

2. The reduced master equation

By introducing two complex parameters

$$f=|f|e^{-i\varphi},\quad g=|g|e^{-i\phi},\quad |f|^2+|g|^2=1, \tag{4}$$

and noting $|M|^2=N(N+1)$, we can identify

$$\begin{aligned} N &\equiv \sinh^2\varphi=\frac{4|f|^2|g|^2}{(|f|^2-|g|^2)^2},\quad N+1=\cosh^2\varphi=\frac{1}{(|f|^2-|g|^2)^2},\\ M &\equiv e^{-i\theta}\sinh\varphi\cosh\varphi=\frac{2fg}{(|f|^2-|g|^2)^2}e^{-i(\varphi+\phi)}, \end{aligned} \tag{5}$$

where $\theta=\varphi+\phi$.

Substituting (4) and (5) into (3) we see that the latter can be re-combined according to the sequence of Σ in each term, i.e.,

$$\begin{aligned} \frac{d\rho}{dt} &= \frac{\lambda}{2(|f|^2-|g|^2)^2}\left[-(a^{\dagger}a\rho-2a\rho a^{\dagger}+\rho a^{\dagger}a)\right.\\ &\quad -4|f|^2|g|^2(aa^{\dagger}\rho-2a^{\dagger}\rho a+\rho aa^{\dagger})\\ &\quad +2|f||g|e^{-i\theta}(a^2\rho-2a\rho a+\rho a^2)\\ &\quad \left.+2|f||g|e^{i\theta}(a^{\dagger 2}\rho-2a^{\dagger}\rho a^{\dagger}+\rho a^{\dagger 2})\right]\\ &= \frac{\lambda}{2(|f|^2-|g|^2)^2}[2(a-2|f||g|e^{-i\theta}a^{\dagger})\rho(a^{\dagger}-2|f||g|e^{i\theta}a)\\ &\quad -(a^{\dagger}-2|f||g|e^{i\theta}a)(a-2|f||g|e^{-i\theta}a^{\dagger})\rho\\ &\quad -\rho(a^{\dagger}-2|f||g|e^{i\theta}a)(a-2|f||g|e^{-i\theta}a^{\dagger})]. \end{aligned} \tag{6}$$

Further, we can simplify (6) as

$$\frac{d\rho}{dt}=\frac{\lambda}{2}(2A\rho A^{\dagger}-A^{\dagger}A\rho-\rho A^{\dagger}A). \tag{7}$$

by defining

$$\begin{aligned} A &\equiv \frac{1}{|f|^2-|g|^2}(a-2|f||g|e^{-i\theta}a^{\dagger})=a\cosh\gamma-a^{\dagger}e^{-i\theta}\sinh\gamma,\\ A^{\dagger} &= \frac{1}{|f|^2-|g|^2}(a^{\dagger}-2|f||g|e^{i\theta}a)=a^{\dagger}\cosh\gamma-ae^{i\theta}\sinh\gamma, \end{aligned} \tag{8}$$

where γ is a squeezing parameter and $[A,A^{\dagger}]=1$.

3. A new entangled state representation

Takahashi and Umezawa in [18] introduced thermo field dynamics (TFD) to convert the statistical average at nonzero temperature T into equivalent pure state expectation value at the expense of introducing an auxilary freedom. For each real field space H they introduced a fictitious field (or a so-called tilde-conjugate field) $\widetilde{H}$. Thus the real vacuum state $|0\rangle$ in H has been doubled to $|0\tilde{0}\rangle$ in $H\otimes\widetilde{H}$. Similarly, the creation operator $a^{\dagger}$ and annihilation operator a acting on H, are accompanied with $\widetilde{a}$ (or $\widetilde{a}^{\dagger}$) in $\widetilde{H}$. In this section, we introduce a new entangled state $|\eta\rangle_c$

$$|\eta\rangle_c=\exp\left[-\frac{1}{2}|\eta|^2+\eta A^{\dagger}-\eta^*\widetilde{A}^{\dagger}+A^{\dagger}\widetilde{A}^{\dagger}\right]|0\tilde{0}\rangle \tag{9}$$

where $|0\tilde{0}\rangle=|0\rangle\otimes|\tilde{0}\rangle$, $|\tilde{0}\rangle$ is annihilated by $\widetilde{A}$, $[A,A^{\dagger}]=[\widetilde{A},\widetilde{A}^{\dagger}]=1$, $\widetilde{A}$ mode is a fictitious mode representing the effect of environment, and the fictitious mode is a counterpart mode of the system-mode A, and

$$\begin{pmatrix} A & \widetilde{A}\\ A^{\dagger} & \widetilde{A}^{\dagger}\end{pmatrix}=\begin{pmatrix}\cosh\gamma & -e^{-i\theta}\sinh\gamma\\ -e^{-i\theta}\sinh\gamma & \cosh\gamma\end{pmatrix}\begin{pmatrix} a & \widetilde{a}\\ a^{\dagger} & \widetilde{a}^{\dagger}\end{pmatrix}. \tag{10}$$

Acing A, $A^{\dagger}$, $\widetilde{A}$, and $\widetilde{A}^{\dagger}$ on the state $|\eta\rangle_c$, we have

$$(A-\widetilde{A}^{\dagger})|\eta\rangle_c=\eta|\eta\rangle_c,\qquad (A^{\dagger}-\widetilde{A})|\eta\rangle_c=\eta^*|\eta\rangle_c, \tag{11}$$

which $|\eta\rangle_c$ is the eigenvector of two commutative operators $(A-\widetilde{A}^{\dagger})$ and $(A^{\dagger}-\widetilde{A})$, and implies operators $(A-A^{\dagger})$ and $(A^{\dagger}-A)$ can be replaced by number η and η^*.

When $\eta=0$, Eq. (9) becomes

$$|\eta=0\rangle_c=\exp(A^{\dagger}\widetilde{A}^{\dagger})|0\tilde{0}\rangle\equiv|I\rangle_c. \tag{12}$$

By introducing the displacement operator $D(\eta)=\exp(\eta A^{\dagger}-\eta^*A)$, we have

$$|\eta\rangle_c=D(\eta)|\eta=0\rangle_c. \tag{13}$$

It is easy to see that $|\eta=0\rangle_c$ has the properties

$$\begin{aligned} A|\eta &= 0\rangle_c=A^{\dagger}|\eta=0\rangle_c,\qquad A^{\dagger}|\eta=0\rangle_c=A|\eta=0\rangle_c,\\ &(A^{\dagger}A)^n|\eta=0\rangle_c=(A^{\dagger}A)^n|\eta=0\rangle_c. \end{aligned} \tag{14}$$

For any general density operators may be also expressed as

$$\rho=\sum_{m,n=0}^{\infty}\rho_{n,m}A^{\dagger m}A^n, \tag{15}$$

one can see that

$$|\rho\rangle\equiv\rho|\eta=0\rangle_c=\sum_{m,n=0}^{\infty}(\rho_{n,m}^*A^{\dagger m}A^n)|\eta=0\rangle_c. \tag{16}$$

Thus the mixed state ρ can be converted into a pure state $|\rho\rangle$ at expense of enlarging the freedom of mode. Due to Eq. (11), we have

$$\begin{aligned} {}_c\langle\eta|\widetilde{A}|\rho\rangle &= -\left(\frac{\partial}{\partial\eta}+\frac{\eta^*}{2}\right){}_c\langle\eta|\rho\rangle,\qquad {}_c\langle\eta|A|\rho\rangle=\left(\frac{\partial}{\partial\eta^*}+\frac{\eta}{2}\right){}_c\langle\eta|\rho\rangle,\\ {}_c\langle\eta|\widetilde{A}^{\dagger}|\rho\rangle &= \left(\frac{\partial}{\partial\eta^*}-\frac{\eta}{2}\right){}_c\langle\eta|\rho\rangle,\qquad {}_c\langle\eta|A^{\dagger}|\rho\rangle=-\left(\frac{\partial}{\partial\eta}-\frac{\eta^*}{2}\right){}_c\langle\eta|\rho\rangle. \end{aligned} \tag{17}$$

Thus we may convert the master equation of ρ into the equation obeyed by ${}_c\langle\eta|\rho\rangle$. This converting process is concise due to the well-behaved properties of the new constructed state ${}_c\langle\eta|$.

4. The solution to the master equation for damping harmonic oscillator

Here we adopt a new approach, acting $|I\rangle_c$ in Eq. (7) and using Eq. (14), we have

$$\frac{d}{dt}|\rho\rangle=\frac{\lambda}{2}(2A\rho A^{\dagger}-A^{\dagger}A\rho-\rho A^{\dagger}A)|\eta=0\rangle_c=\frac{\lambda}{2}(2A\widetilde{A}-A^{\dagger}A-\widetilde{A}^{\dagger}\widetilde{A})|\rho\rangle. \tag{18}$$

Thus we have converted the master equation in Eq. (7) into an evolution equation of $|\rho\rangle$. Its formal solution is