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Propagation of Cartesian beams in the non-Kerr law media with power law nonlinearity

G. Honarasa *, Z. Farmani, A. Keshavarz

Department of Physics, Shiraz University of Technology, Shiraz, Iran

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ABSTRACT

In this paper, propagation of the Cartesian beams in the non-Kerr law media with power law nonlinearity is investigated by ABCD matrix method. For this purpose, the transfer matrix and critical power of the Cartesian beams for propagating through these media are presented. Then, the evolutions of the beam width and curvature radius of the Cartesian beams during propagation are analyzed.

1. Introduction

The Cartesian beams are the general solution of the paraxial wave equation in Cartesian coordinates [1]. The complex amplitude of the Cartesian beams is described as confluent hypergeometric functions or parabolic cylinder functions and these beams are characterized by three complex parameters. For special values of the beam parameters, the Cartesian beams convert to many known solutions of optical beams such as the standard, elegant and generalized Hermit–Gauss beams, Cosine–Gauss beams, the Lorentz beams, the Airy beams and the fractional order beams [2–6]. Propagation of the Cartesian beams in ABCD systems and nonlocal nonlinear media has been investigated [1,7]. Bandres et al. introduced a closed-form expression for the overlap between two different Cartesian beams [5]. Propagating and shaping characteristics of the one-dimensional Cartesian Parabolic-Gaussian beams through complex ABCD optical systems have been studied by Lopez-Mago et al. [8]. The main nonlinear effect that arises in optical materials is the Kerr effect. Recently, nonlinear effects of non-Kerr law media which can be found in various materials have been received attention extensively [9–15]. There are several non-Kerr law nonlinearities such as power law, parabolic law, the dual-power law, and exponential law. Although, propagation of the optical solitons in non-Kerr law media has been simulated by solving nonlinear Schrodinger equation numerically [16–19], in this paper, propagation of the Cartesian beams in the non-Kerr law media with power law nonlinearity is simulated by using suitable transfer matrix.

The paper is organized as follows: In Section 2, the propagation of the Cartesian beam in an arbitrary ABCD optical system is briefly reviewed. The transfer matrix of non-Kerr law media with power law nonlinearity is defined in Section 3. propagation properties of Cartesian beam is investigated in Section 4. Finally, the paper is concluded in Section 5.

2. Cartesian beam

The transverse distribution of the even and odd Cartesian beams at the output plane of an ABCD optical system is described by the confluent hypergeometric function ${}_1F_1(a, b; x)$ as follows [1]:

$$ {}_tU_\beta(x;p,q) = {}_t\zeta_\beta (Px^2)^{(t-1/2)/2}\, {}_1F_1(\beta, t; Px^2) \exp\left(\frac{ikx^2}{2q}\right). \quad (1)$$

Since the fields can be split into products of solutions in the x and y; ${}_tU_\beta(x, y; p, q) = {}_tU_\beta(x; p, q)\, {}_tU_\beta(y; p, q)$, without loss of generality, the solution of the paraxial wave equation in only one Cartesian coordinate has been considered. The value of parity factor t for even beams is 1/2 and for odd beams is 3/2. In Eq. (1), k is the wavenumber, β is a complex parameter and

$$P \equiv P(p,q) = \frac{ik}{2}\left(\frac{1}{p} - \frac{1}{q}\right). \quad (2)$$

The Cartesian beams are characterized by the complex parameters p and q which are related to their initial values (p_0, q_0) at input plane $z = 0$ by the following transformation relations:

$$p = \frac{Ap_0 + B}{Cp_0 + D}, \quad q = \frac{Aq_0 + B}{Cq_0 + D}. \quad (3)$$

* Corresponding author. Tel.: +98 71 37261392.
E-mail addresses: honarasa@sutech.ac.ir (G. Honarasa), z.farmani@sutech.ac.ir (Z. Farmani), keshavarz@sutech.ac.ir (A. Keshavarz).

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The factor ${}_t\zeta_\beta$

$$ {}_t\zeta_\beta = \frac{(A+B/q_0)^{\beta-(t+1/2)/2}}{(A+B/p_0)^{\beta-(t-1/2)/2}} \quad (4)$$

is a scaling factor and comes from the propagation of the beam through the ABCD system. Eq. (1) reduces to many known solutions of optical beams with rectangular symmetry for some special values of the three parameters (β, p_0, q_0) that are complex in the most general case. For example, the elegant Hermite–Gaussian beams are obtained by setting $p_0=\infty$, $q_0=-iz_R$ and $\beta=-n$ where $z_R=k\omega_0^2/2$ is Rayleigh distance of the Gaussian beam and n is an integer number [1,5]. The propagation of the Cartesian beams through any optical system with ABCD transfer matrix can be analyzed by Eqs. (1)–(4).

3. Non-Kerr law media with power law nonlinearity

Several forms of nonlinearity have been investigated by developing of the Kerr media. The most important form of these nonlinearities is power law nonlinearity. This nonlinearity is a generalization of Kerr law nonlinearity and can be found in various materials such as nonlinear plasmas and semiconductors. Refraction index of the non-Kerr law media due to power law nonlinearity is given by [9]:

$$ n = n_0 + n_{2E}|E|^{2m}, \quad (5)$$

where n_0 is linear refractive index of the medium, n_{2E} is the higher order nonlinear refractive index, E is the electric filed and m is the order of power law nonlinearity. For special case $m=1$, the power law nonlinearity reduces to Kerr law nonlinearity. In (5), it is necessary to have $0<m<2$ to prevent wave collapse [9,20] and $m \neq 2$ to avoid self-focusing singularity [20]. By considering a Gaussian field distribution for the electric field of incident beam $E=E_0\exp(-r^2/\omega_0^2)$ and expanding the exponential function and keeping the two first terms, Eq. (5) becomes

$$ n = (n_0 + n_{2E}E_0^{2m}) - 2mn_{2E}E_0^{2m}(r^2/\omega_0^2) \approx n_0 - \frac{2mn_{2E}E_0^{2m}}{\omega_0^2}r^2, \quad (6)$$

where $n_{2E}E_0^{2m} \ll n_0$ which is true in practice has been used.

On the other hand, transfer matrix for beam propagation in lens-like media with following refractive index

$$ n = n_0 - \frac{n_2}{2n_0}r^2 \quad (7)$$

is expressed by [21]:

$$ \begin{pmatrix} \cos\left(\frac{z\sqrt{n_2}}{n_0}\right) & \frac{n_0}{\sqrt{n_2}}\sin\left(\frac{z\sqrt{n_2}}{n_0}\right) \\ -\frac{\sqrt{n_2}}{n_0}\sin\left(\frac{\sqrt{n_2}}{n_0}\right) & \cos\left(\frac{z\sqrt{n_2}}{n_0}\right) \end{pmatrix}. \quad (8)$$

By comparing Eqs. (6) and (7), one find that the transfer matrix (8) can be used for non-Kerr law media with power law nonlinearity considering n_2 as follows:

$$ n_2 = \frac{4mn_0n_{2E}E_0^{2m}}{w_0^2}. \quad (9)$$

Then, the convergence angle of a Gaussian beam in a non-Kerr law media with power law nonlinearity can be expressed by [22]:

$$ \theta_{NL} = \frac{2\omega_0\sqrt{n_2}}{\pi n_0}. \quad (10)$$

Also, the angle of spread of the beam in absence of any nonlinear effects due to the diffraction is given by [23]:

$$ \theta_{SP} = \frac{\lambda_0}{\pi\omega_0 n_0}. \quad (11)$$

For $\theta_{SP}=\theta_{NL}$, self-focusing and diffraction will cancel each other. In this case, using Eq. (9) the critical electric field of Gaussian beam is obtained as follows:

$$ E_{cr}^{GB} = \left(\frac{\lambda_0}{4\omega_0\sqrt{mn_0n_{2E}}}\right)^{1/m} \quad (12)$$

and the critical power of the Gaussian beam is given by [22]:

$$ \mathcal{P}_{cr}^{GB} = \frac{1}{4}\pi cn_0\varepsilon_0\omega_0^2|E_{cr}|^2 = \frac{\pi cn_0\varepsilon_0\omega_0^2}{4}\left(\frac{\lambda_0}{4\omega_0\sqrt{mn_0n_{2E}}}\right)^{2/m}. \quad (13)$$

If the power of incident Gaussian beam $\mathcal{P}_0$ is equal to the critical power $\mathcal{P}_0=\mathcal{P}_{cr}^{GB}$, the Gaussian beam propagates without any focusing or defocusing in the non-Kerr media with power law nonlinearity. The critical power of the Cartesian beam will be defined in Section 4. If the input power is equal to the critical power of the Cartesian beam, the beam width of the Cartesian beam keeps invariant during propagation through the non-Kerr media with power law nonlinearity.

By using Eqs. (9) and (13), the transfer matrix (8) for the non-Kerr media with power law nonlinearity can be described by

$$ \begin{pmatrix} \cos\left[\frac{\pi z(\mathcal{P}_0/\mathcal{P}_{cr}^{GB})^{m/2}}{2n_0z_R}\right] & \frac{2n_0z_R}{\pi(\mathcal{P}_0/\mathcal{P}_{cr}^{GB})^{m/2}}\sin\left[\frac{\pi z(\mathcal{P}_0/\mathcal{P}_{cr}^{GB})^{m/2}}{2n_0z_R}\right] \\ -\frac{\pi(\mathcal{P}_0/\mathcal{P}_{cr}^{GB})^{m/2}}{2n_0z_R}\sin\left[\frac{\pi z(\mathcal{P}_0/\mathcal{P}_{cr}^{GB})^{m/2}}{2n_0z_R}\right] & \cos\left[\frac{\pi z(\mathcal{P}_0/\mathcal{P}_{cr}^{GB})^{m/2}}{2n_0z_R}\right] \end{pmatrix}. \quad (14)$$

4. Propagation properties of the Cartesian beams

Now, the dynamical evolution of Cartesian beams in non-Kerr law media with power law nonlinearity can be studied based on Eq. (1) and the transfer matrix (14).

Fig. 1 shows the transverse field distribution of a typical Cartesian beam during propagation through the non-Kerr media with power law nonlinearity for different values of input powers. In Fig. 2, the behavior of a typical Cartesian beam in $x-z$ and $y-z$ planes have been plotted. It can be found from the figures that when the Cartesian beam propagates these media, the intensity distribution varies periodically. The periodic distance is $z=2n_0z_R/(\mathcal{P}_0/\mathcal{P}_{cr}^{GB})^{m/2}$ which can be found from Eq. (14). In both figures (and in the rest of the paper), parameters of the Cartesian beam in plane $z=0$ set as $p_0=\infty$, $q_0=-iz_R$, $\beta_x=-1$, $\beta_y=-0.5$, $t_x=1/2$ and $t_y=3/2$ where $z_R=k\omega_0^2/2$ is Rayleigh distance of the Gaussian beam. With these parameters the Cartesian beam reduces to an Elegant Hermit–Gaussian beam [1,5].

The second-order moments definition can be used to calculate the beam width of the Cartesian beams. The beam width at the waist plane w_1 can be obtained by [24]:

$$ w_1^2 = 4\frac{\int_{-\infty}^{\infty}x^2|{}_tU_\beta(x;p_0,q_0)|^2dx}{\int_{-\infty}^{\infty}|{}_tU_\beta(x;p_0,q_0)|^2dx} \quad (15)$$

and the beam width at the observation plane w_2 after propagation through an optical ABCD system is related to w_1 as follows:

$$ w_2^2 = A^2w_1^2 + 2ABV_1 + B^2U_1, \quad (16)$$