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# Effect of the phase damping of two qubits on both the quantum discord and non-local correlation

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ABSTRACT

An analytical solution of the master equation for two qubits-field system, in the dispersive regime, are investigated. The qubits are initially in Werner states and the field in coherent state. Under the influence of the damping, the geometric measure of quantum discord (GMQD) and the measurement-induced nonlocality (MIN) are investigated. GMQD and MIN are compared and illustrated their different characteristics. It is found that under the influence of damping the phenomenon of the death occurs for GMQD, but this phenomenon does not occur for MIN even when the damping parameter is high. The initial conditions for the qubits play an important role in the phenomenon of collapses and revivals for GMQD and MIN.


## 1. Introduction

Quantum entanglement (QE) is one of the most remarkable features of quantum mechanics and it plays a central role in quantum information and communication theory [1]. There exists, however, nonclassical correlation, which is more general and more fundamental than entanglement in the sense that separable mixed states can have nonclassical correlation [2,3]. Moreover, nonclassical correlation other than entanglement can be responsible for the quantum computational efficiency of deterministic quantum computation with one qubit. Quantum correlation arises from noncommutativity of operators representing states, observables, and measurements [4]. QE which refers to the separability of the states, is very important in quantum information processing and can be realized in many kinds of physical systems which involve quantum correlation. An alternative classification for quantum correlations, which is based on quantum measurements, has arisen in recent years and also plays an important role in quantum information theory [4–6].

In particular, quantum discord (QD) [2] is introduced to measure these quantum correlations. There exist indeed separable mixed states having nonzero discord and the separable mixed states can be used to perform useful quantum tasks [7]. Evaluation of QD in general requires considerable numerical minimization and analytical expressions are known only for certain classes of states. The authors in [8] evaluated analytically the QD for a large family of two-qubit states, and make a comparative study of the relationships between classical and quantum correlations in terms of the QD.

The authors in [9] propose a geometrical way of quantifying QD, which is termed as GMQD. GMQD can be extended to any number of subsystems, though evaluating the measure of discord becomes progressively more difficult with the increasing of the number of subsystems and that of their dimensionality. Moreover, the authors in [10] evaluate GMQD for an arbitrary state and obtain an explicit and tight lower bound.

Very differently, MIN [11] has been proposed to interpret the maximum global effect caused by locally invariant measurements, the authors claim that MIN is in some sense dual to GMQD. Anyway, both GMQD and MIN may be considered to be the measurement tool of quantum correlation. The interaction of a quantum system with its environment causes the rapid destruction of crucial quantum properties and drives the system to an incoherent state.

It was shown by the authors in [12] that entanglement of a bipartite system decays to zero in a finite time, which is called entanglement sudden death (ESD), while coherence vanishes exponentially with time to zero. Subsequently, ESD in different systems has been made by various groups [13–16]. Another important situation is the dephasing environment, in which energy transfer from the system to the environment does not occur. Some work has been devoted to this issue [17,18]. Recently, by using carefully engineered interactions between system and environments, experimental studies have been carried out to demonstrate ESD, and ESD

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has been observed both in photons [19] and in atomics ensembles [20]. By using GMQD and MIN, the quantum correlation of two qubits is studied in [21,22]. While the quantum correlations and QD with and GMQD is studded in [23]. Also, the pairwise correlations, including QD and GMQD is studded in [24]. The quantumness of the correlations between two qubits by using measurement-induced disturbance is studded [25–27].

Motivated by these premises, here we focus on studying the quantum correlation GMQD and MIN for two qubits-field system in the dispersive reservoir and illustrate their different characteristics. The organization of the article runs as follows: Section 2 presents the model and the dispersive limit is taken and the solution of the master equation is presented. The dynamics of GMQD and MIN are considered for two different initial states of the qubits in Section 3. Followed by discussion of numerical results in Section 4. In Section 5 we draw our conclusions.

## 2. Tow-qubit model system

We start the investigation by describing the system under consideration and giving the basic relations and equations, which will be frequently used in this paper. We consider here two-qubit- field system in the dispersive regime with a phase damping reservoir. The two qubits considered here can be impurity atoms in a photonic crystal or single-mode fiber, or atoms in a magnetic trap or micro-cavity, or atoms in other man-made environments. All of these systems can be accurately described by models of collective decay by interaction with a common heat bath, which can be reduced to models of atom dynamics in one-dimensional resonant fields. Two initially interacting qubits, labeled by $A$ and $B$, are chosen as the model. A two-level qubit with an excited state $|1\rangle$ and a ground state $|0\rangle$ is considered. The unavoidable interaction between the system and the environment will lead to the loss of coherence, and may also lead to the loss of entanglement and nonclassical correlation. From the quantum master equation:

$$\frac{d\rho}{dt} = i[\rho, \hat{H}] + Ł\rho, \tag{1}$$

where the phase damping reservoir is described by:

$$Ł\rho = \frac{\xi_1}{2}(\sigma_z^{(A)}\rho\sigma_z^{(A)} - \rho) + \frac{\xi_2}{2}(\sigma_z^{(B)}\rho\sigma_z^{(B)} - \rho) \tag{2}$$

Here, the Hamiltonian $H$ of the two-qubit system interacting with the filed in the dispersive regime is given by [28]:

$$\begin{aligned}\hat{H} &= \lambda[\sum_{i=A,B}\{|1\rangle_{ii}\langle 1|\hat{a}\hat{a}^\dagger - |0\rangle_{ii}\langle 0|\hat{a}^\dagger\hat{a}\} + (|1\rangle_{11}\langle 0| \otimes |0\rangle_{22}\langle 1| \\ &\quad + |0\rangle_{11}\langle 1| \otimes |1\rangle_{22}\langle 0|)],\end{aligned} \tag{3}$$

where $\hat{a}^\dagger$ ($\hat{a}$) is the creation (annihilation) operator and the two eigenstates of the individual qubit ($|0\rangle$,$|1\rangle$) constitute the qubit states and $\lambda$ is the effective interaction constant. Parameters $\xi_1$ and $\xi_2$ are the phase damping constants for the two qubits, $\sigma_z^{(i)} = |1\rangle_{ii}\langle 1| - |0\rangle_{ii}\langle 0|$, $i = A, B$.

The density matrix $\rho$, which describes the state of the system, can be evaluated. From the state, the dynamics of GMQD between the qubits can be investigated. In the expression, $Ł\rho$ includes the effect of the interaction between the environment and the qubits. In the following, the correlated dissipative environments will be investigated. The master equation (Eq. (1)) can be solved to obtain $\rho_{ij}(t)$, $(i, j = 1, 2, 3, 4)$ (for simplicity we will take $\xi_1 = \xi_2 = \xi$). To do this, we suppose the qubits and field are initially in the form:

$$\rho(t) = \sum_{m,n=0} q_n q_m |m\rangle\langle n| \otimes \rho^{AB}(0), \tag{4}$$

with $q_n = e^{\frac{-|\alpha|^2}{2}}\frac{\alpha^n}{\sqrt{n!}}$ and $\alpha$ is a complex number.

Where the field is initially in coherent state $|\alpha\rangle$. While the qubits are initially in Werner states defined by [29]:

$$\rho^{AB}(0) = \mu|\varphi\rangle\langle\varphi| + \frac{1}{4}(1-\mu)I \tag{5}$$

where $\mu$ is a real number, which indicates the purity of initial states, $I$ is the identity matrix. We calculate the reduced density matrix $\rho^{AB}$ of two qubits by tracing out the field variables in two cases.

Case 1. We consider the initial state $|\varphi\rangle = sin\theta|11\rangle + cos\theta|00\rangle$. Therefore, the reduced density matrix of the two qubits $\rho^{AB}(t)$ is given by:

$$\rho^{AB}(\tau) = \begin{pmatrix} \frac{1}{4}(1-\mu)+\mu\sin^2\theta & 0 & 0 & \rho_{14}(\tau) \\ 0 & \frac{1}{4}(1-\mu) & 0 & 0 \\ 0 & 0 & \frac{1}{4}(1-\mu) & 0 \\ \rho_{41}(\tau) & 0 & 0 & \frac{1}{4}(1-\mu)+\mu\cos^2\theta \end{pmatrix}, \tag{6}$$

where $\rho_{14}(\tau) = \rho_{41}^*(\tau) = \frac{1}{2}\mu\sin 2\theta\mathbf{e}^{-2\tau(\gamma+i)-|\alpha|^2(1-e^{-4i\tau})}$.

Case 2. If the initial state is considered to be$|\varphi\rangle = sin\theta|10\rangle + cos\theta|01\rangle$, then the reduced density matrix of the qubits read as:

$$\rho^{AB}(\tau) = \begin{pmatrix} \frac{1}{4}(1-\mu) & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \frac{1}{4}(1-\mu) \end{pmatrix}, \tag{7}$$

where,

$$\begin{aligned}\rho_{22}(\tau) &= \frac{1}{4}[1+\mu-2\mu e^{-\gamma\tau}\cos 2\theta(\cos\sqrt{4-\gamma^2}\tau + \frac{2\gamma\sin\sqrt{4-\gamma^2}\tau}{\sqrt{4-\gamma^2}})], \\ \rho_{23}(\tau) &= \rho_{32}^*(\tau) = [\frac{-2\mu i\cos 2\theta\sin\sqrt{4-\gamma^2}\tau}{\sqrt{4-\gamma^2}} + \mu\sin 2\theta e^{-\gamma\tau}]e^{-\gamma\tau}, \\ \rho_{33}(\tau) &= \frac{1}{4}[1+\mu+2\mu e^{-\gamma\tau}\cos 2\theta(\cos\sqrt{4-\gamma^2}\tau + \frac{2\gamma\sin\sqrt{4-\gamma^2}\tau}{\sqrt{4-\gamma^2}})]\end{aligned}$$

with $\tau = \lambda t$ and $\gamma = \frac{\xi}{\lambda}$.

These solutions (6) and (7) are used in the following section to study the dynamics of geometric measure of quantum discord and measurement-induced nonlocality of two qubits-field system in the dispersive reservoir.

## 3. Dynamics of GMQD and MIN

In this section, the dynamics of GMQD and MIN are considered. Geometric measure of quantum discord is introduced by Ref. [30], which measures the quantum correlations through the minimum Hilbert–Schmidt distance between the given state and zero discord state. Generally, GMQD is defined as [31]

$$Q_A(\rho^{AB}) = \min\|\rho^{AB} - \Im\|^2, \tag{8}$$