

Odd symmetrical square-root phase mask to extend the depth of field in wavefront-coded imaging systems



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ABSTRACT

Extend-the-depth-of-field imaging can be achieved by wavefront coding technique, which inserts an odd symmetrical phase mask into the pupil plane to generate the invariant imaging property over a wide range of defocus. Obtaining the low sensibility to defocus for wavefront coding systems depends on designing a proper phase profile. Here we proposed a phase mask combining the power function with the square-root function to maximize the invariant imaging property to defocus. Judging by defocused modulation transfer function (MTF) and Hilbert space angle, numerical comparisons with the cubic mask, exponential mask and improved logarithmic mask prove the superiority of proposed square-root phase mask in extending the depth of field.

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1. Introduction

Theoretical framework of wavefront coding emerged from the analysis of ambiguity function representation of defocused optical transfer function (OTF), accompanying that the first solution of the ambiguity function equation is an odd symmetrical cubic phase function, which can be used to extend the depth of field of incoherent imaging systems [1]. The imaging processes of wavefront coding systems include two steps: the image sampling step referred to that an odd symmetrical phase mask encodes the wavefront to capture the similarly intermediate images at different position of defocus, and the image restoration step referred to that the intermediate images are subsequently decoded using a deconvolution filter to obtain the final high-quality images.

The deconvolution kernel for wavefront coding systems is its encoded point spread function (PSF) or OTF, therefore, an optimized principle of systems is to minimize difference between in-focus PSF and defocused PSFs so that all intermediate images can easily be deblurred through only one deconvolution filter. The similarity of PSFs or OTFs depends on the phase profile of phase mask. Several kinds of phase masks, including cubic mask [1], exponential phase mask [2], logarithmic phase mask [3–5], sinusoidal phase mask [6], polynomial phase mask [7], rational mask [8] and high-order phase mask [9], have been proposed. We also propose a tangent phase mask to obtain defocus-invariant

modulation transfer function (MTF) within a wide range of defocus in [10]. The performance evaluations of these phase masks are presented by using several evaluation methods, such as Fisher Information (FI), Hilbert space angle and Strehl Ratio (SR).

Among them the cubic mask is first odd-symmetrical phase mask and its exact analytical representation has been used to display the imaging properties of wavefront coding systems, logarithmic mask with several improved shapes and exponential mask are the other two popular phase masks. Two-dimensional rectangularly separable phase functions of these three phase masks can be respectively described as,

$$f_{\text{Cubic}}(x, y) = ax^3 + ay^3, \quad (1)$$

$$f_{\text{Exponential}}(x, y) = ax \exp(bx^2) + ay \exp(by^2), \quad (2)$$

$$f_{\text{Improved logarithmic}}(x, y) = \text{sgn}(x)ax^4 \log(|x| + b) + \text{sgn}(y)ay^4 \log(|y| + b), \quad (3)$$

where a and b are the phase mask parameters which control the magnitude of phase deviation, both x and y are the normalized pupil plane coordinates and $\text{sgn}(\cdot)$ represents the sign function which is defined as 1 for $x > 0$ and -1 for $x < 0$. In addition Eq. (3) presents an optimal phase profile in all logarithmic phase masks.

By investigating these phase masks, it is no hard to find that they have a uniform characteristic which the phase profiles bend toward the pupil plane. However, the rational phase mask presents a reverse characteristic which its phase profile bends away from the pupil plane. It seems that the bends' orientations of phase profiles is

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symmetrical with respect to the plane, $f(x, y) = px + py$ (The parameter p control the slope of the plane $f(x, y)$ for making the plane $f(x, y)$ locate between the rational mask and the other masks). Ref. [8] has shown the superiority of rational phase mask in reducing the sensitivity to defocus. But, the additional phase masks like the rational mask's phase profile have not been reported until now, although the rational phase mask show better performance in the defocus-invariant imaging property. In this Letter, we propose a novel odd symmetrical phase mask by combining the power function and the square root function. The contour of proposed mask resembles the rational phase mask. Through comparison with previous phase mask, numerical results demonstrate the superiority of square-root phase mask in achieving defocus-invariant imaging characteristics.

2. Square-root phase mask with optimization parameter

Our proposed square-root phase mask can be represented as

$$f_{\text{Square-root}}(x, y) = ax\sqrt{1-x^2} + ay\sqrt{1-y^2}. \quad (4)$$

Above the square-root mask only includes a phase parameter to control its phase profile. In order to carry out the reasonable performance comparison, first the phase mask parameters should be optimized under the same condition. For the wavefront-coded imaging system, the optimized phase parameter is to balance the imaging performance between the similarity and the recoverability. The similarity represents similar degree of the encoded images and the recoverability indicates if easy to deblur the encoded images. Generally, the merit functions of phase mask parameters should make the imaging properties at different defocus position as similar as possible under a same constraint condition, which assure the recoverability of encoded images. This, the optimization process is restricted to the case of one-dimensional wavefront coding system for mathematical simpleness without loss of generality. We establish the merit function based on FI matrix mentioned above can be expressed as

$$\left\{ \min \left\{ \int_{-\psi_{\max}}^{\psi_{\max}} \int_{-1}^1 |H(u, \psi)| \left[\frac{\partial}{\partial \psi} \ln |H(u, \psi)| \right] \left[\frac{\partial}{\partial \psi} \ln |H(u, \psi)| \right]^T du d\psi \right\} \right. \\ \left. \text{subject to } \int_{-1}^1 |H(u, \psi=0)| du \geq \sigma \right\} \quad (5)$$

where the function H is the normalized OTF, u is the normalized spatial frequency, $[\cdot]^T$ denotes the transpose matrix, σ is the minimum acceptable integral area of MTF, and ψ is the defocus parameter, can be defined as

$$\psi = \frac{\pi L^2}{4\lambda} \left(\frac{1}{f'} - \frac{1}{d_0} - \frac{1}{d_i} \right) \quad (6)$$

where L is the aperture diameter, λ is the wavelength of light, and f' , d_0 , and d_i , are the focal length, the object distance, and the image distance, respectively.

In this letter, we optimize the phase parameters whose the phase functions are given by Eqs. (1)–(4) based on the merit function of Eq. (5) and the genetic algorithm is adopted to solve this constrained nonlinear optimization problem. The defocus parameter ψ is set as 0, 6, 12, 18, 24, and 30 and 256×256 dimensional FI matrix is used in the optimization process. According to Eq. (5), the values of the phase mask parameter after optimization are listed in Table 1 when the constraint parameter $\sigma = 0.32, 0.35$ are considered in the normalized pupil coordinate. Fig. 1 shows the phase contours

Table 1
Optimum parameters of each phase masks*.

σ	Cubic (a)	Exponential (a, b)	Logarithmic (a, b)	Square-root (a)
0.32	78.82	33.68, 1.49	140.79, 1.81	117.46
0.35	66.58	28.45, 1.47	242.66, 0.95	97.93

* The unit of a is radians and b has no unit.

of cubic mask, improved logarithmic mask, exponential mask and square-root mask with the optimized parameters correspond to σ equaling 0.32 in Table 1.

As shown in Fig. 1, the square-root phase mask presents a quite different profile which bends away from the pupil plane mentioned above. This phase profile will result in an excellent defocus-invariant imaging characteristic in following depiction.

3. Results and discussion

With the data in Table 1, defocused MTF curves of four phase masks are shown in Fig. 2 when the defocus parameter ψ is set as 0, 6, 12, 18, 24, and 30, respectively. As Fig. 2 shows, compared with the conventional optical system, the magnitudes of defocused MTF curves for four phase masks have a considerable reduction but produce a nearly invariant distribution at all defocus position. This invariant distribution displays the wavefront-coding imaging systems' defocus-invariant property. It is not difficult to find that the cubic phase mask present an obvious oscillations at the high spatial frequency, meanwhile the considerable oscillations of improved logarithmic mask and exponential mask appear in the low spatial frequency. Comparatively, the proposed square-root phase mask presents the reduced oscillation of the defocus MTF curves within all spatial frequency range. This reduced oscillation can result in the invariant MTF at different defocus position, i.e., the square-root phase mask can perform better in the defocus-invariant characteristics.

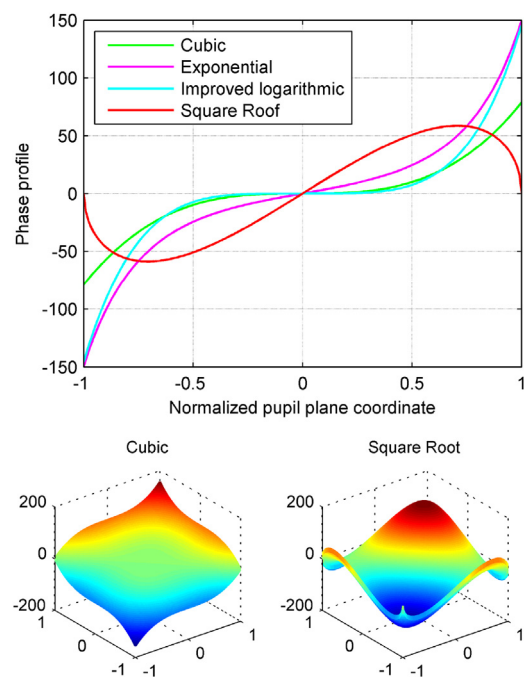


Fig. 1. Phase profiles of cubic, exponential, improved logarithmic, square-root phase masks. Optimized parameters adopted correspond to σ equaling 0.32. One-dimensional profiles are in above image and two-dimensional profiles of cubic and square-root mask are in below images.

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