Optik 127 (2016) 1116–1120

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Quality factors of an aberrant Gaussian beam diffracted through a convex grating spectrometer

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ARTICLE INFO

Article history:
Received 1 February 2015
Accepted 29 October 2015

Keywords:
Complex-source-point
Gaussian beam
Spectrometer
Quality factor
Aberration

ABSTRACT

We analyze the evolution of a complex-source-point spherical wave through a concentric catoptric system that is made up of one convex grating and two concave mirrors. First we trace the paraxial paths of a chief ray propagating from a complex source point through the system to complex image points of different orders. Then we find an expression of the fourth order-corrected Gaussian beam which includes Seidel-type aberrations. Finally, we analyze the quality of an aberrant Gaussian beam diffracted through the system. The results presented here will be useful in dealing with the propagation of a focused Gaussian beam through a diffractive optical system and the beam quality degradation due to aberration of the system.

1. Introduction

A complex-source-point spherical wave (CSPSW) can be expanded near the optical axis in a power series whose first term is a Gaussian beam. The CSPSW was first introduced by Deschamps [1] and extensively analyzed by Felsen [2]. Later, Couture and Belanger showed that the sum of all the higher order corrections to the paraxial Gaussian beam was reduced to the CSPSW [3]. The CSPSW is very useful in dealing with the propagation and scattering of a strongly focused laser beam [4,5]. Recently, we examined the evolution of the CSPSW along the axis of an optical system of spherical (or non-spherical) surfaces so that the fourth order-corrected Gaussian beam could be represented in terms of spherical aberration [6], and we also used the CSPSW to analyze the quality of a slightly inclined Gaussian beam passing through a rotationally symmetric system which involves any combination of refracting (or reflecting) surfaces [7], zone plates [8], and graded-index lenses [9]. On the other hand, the advanced manufacturing techniques to produce diffractive optical surfaces renewed interest in diffractive optics that utilized some fundamental structure periodicity [10,11]. In an earlier work [12], we found the stigmatic condition that could be useful in rapidly creating an initial design of a convex grating spectrometer. However, the propagation of the CSPSW through the system and the beam quality degradation due to aberration of the system have not been analyzed.

In this paper we examine the evolution of the CSPSW through a concentric catoptric system that is made up of one convex grating and two concave mirrors. The system is generally not telecentric in the sense that the aperture stop at the grating may be displaced from the back (or front) focal plane of the preceding (or following) optics. In our approach, we first trace the paraxial paths of a chief ray propagating from a complex source point through the system to complex image points of different orders. Then we derive the wave function of the light converging to the complex image point that can be represented in terms of Seidel-type aberrations, while the terms of up to fourth order in aperture variables are taken into account. Finally, we analyze the quality of an aberrant Gaussian beam diffracted through the system, on condition that the center of the incident Gaussian beam is located in the plane orthogonal to the optical axis, containing the common center of curvature of the system. The results described here will be useful in dealing with the propagation of a focused Gaussian beam through a diffractive optical system and the beam quality degradation due to aberration of the system.

2. Propagation of a Gaussian beam through a convex grating spectrometer

Fig. 1 shows the setup of a concentric catoptric system that is composed of one convex grating and two concave mirrors, in which C is the common center of curvature of the grating and the mirrors, and V is the center of the grating taken as an aperture stop. The aperture stop may be displaced from the back (or front) focal plane of the preceding (or following) optics. The coordinate system

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http://dx.doi.org/10.1016/j.ijleo.2015.10.202
0030-4026/

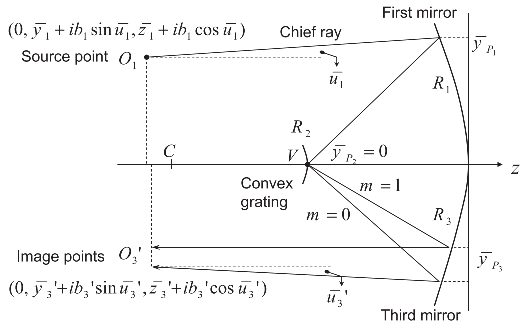

Fig. 1. A concentric catoptric system that is composed of one convex grating and two concave mirrors, in which C is the common center of curvature of the grating and the mirrors, and V is the center of the grating taken as an aperture stop. The coordinate system is referenced to the tangent plane of each mirror which is parallel to the x and y axes, and a straight line connecting the two points C and V is chosen as the z axis. The grating lines ruled on the convex surface are parallel to the x axis and equally spaced with a period of p along the y axis. A chief ray of light starts from a source point O_1 and goes to another point V on the grating surface after being reflected by the first concave mirror. The ray of light diffracted at the point V arrives at different image points O_3' according to diffracted orders after being reflected by the third concave mirror. The chief ray from O_1 (or O_3') placed at the yz plane, makes an angle of $\bar{u}_1$ (or $\bar{u}_3'$) with the z axis. The local coordinates of O_1 (or O_3') referenced to the first (or third) mirror are assumed to be of $(0, \bar{y}_1 + ib_1 \sin\bar{u}_1, \bar{z}_1 + ib_1 \cos\bar{u}_1)$ (or $(0, \bar{y}_3' + ib_3' \sin\bar{u}_3', \bar{z}_3' + ib_3' \cos\bar{u}_3')$).

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A spherical wave radiating from the point O_1 to another point (x_1, y_1, z_1) in the medium of refractive index n_1 may be expressed as

$$\psi(x_1, y_1, z_1, t) = \frac{A}{r_1} \exp(ikn_1 r_1 - i\omega t), \tag{1}$$

where A is the constant, $i(=\sqrt{-1})$ is the imaginary symbol, $k(=2\pi/\lambda)$ is the magnitude of the wave vector in vacuum, ω is the angular frequency of the light, and

$$r_1 = [x_1^2 + (y_1 - \bar{y}_1 - ib_1 \sin\bar{u}_1)^2 + (z_1 - \bar{z}_1 - ib_1 \cos\bar{u}_1)^2]^{1/2}. \tag{2}$$

If we choose the branch of r_1 such that its real part is equal to $(z_1 - \bar{z}_1)$ when it is large, the wave function (1) can then be approximated as

$$\psi(x_1, y_1, z_1, t) \simeq \frac{A}{z_1 - \bar{z}_1 - ib_1} \exp\left\{ ikn_1 \left[z_1 - \bar{z}_1 - ib_1 + \frac{x_1^2 + (y_1 - \bar{y}_1 - ib_1\bar{u}_1)^2}{2(z_1 - \bar{z}_1 - ib_1)} \right] - i\omega t \right\}, \tag{3}$$

in the paraxial regime where $|\bar{u}_1| \ll 1$ and $x_1^2 + (y_1 - \bar{y}_1)^2 \ll (z_1 - \bar{z}_1)^2 + b_1^2$. It is obvious that the wave function (3) can represent a slightly inclined Gaussian beam centered at the local coordinates $(0, \bar{y}_1, \bar{z}_1)$, in which the beam axis makes an angle of $\bar{u}_1$ with the z axis and the Rayleigh range of the beam is given by $b_1 = n_1 \pi w_1^2/\lambda$ in terms of the minimum spot radius w_1 [1–3]. Similar analyses may be done to the case of a spherical wave converging to the image point O_3'. In that case, the spherical wave is reduced to a paraxial Gaussian beam centered at the local coordinates $(0, \bar{y}_3', \bar{z}_3')$, in which $\bar{u}_3'$ and b_3' correspond to the slope of the beam axis and the Rayleigh range of the beam, respectively.

Applying Fermat's principle, we can get the formal equations for tracing the paraxial path of the chief ray from O_1 through the optical system to O_3' as follows [7]:

$$\begin{aligned}
&\frac{n_j'}{\bar{z}_j' + ib_j'} = \frac{n_j}{\bar{z}_j + ib_j} + \frac{n_j' - n_j}{R_j},\\
&\bar{y}_{P_j} = \bar{y}_j - \bar{z}_j \bar{u}_j,\\
&n_j'\bar{u}_j' - \frac{m\lambda}{p_j} = n_j\bar{u}_j - \frac{n_j' - n_j}{R_j}\bar{y}_{P_j},\\
&\bar{y}_j' = \bar{y}_{P_j} + \bar{z}_j'\bar{u}_j',
\end{aligned} \tag{4}$$

for j = 1, 2, and 3 . In Eq. (4) n_j (or n_j') is the refractive index for a ray incident on (or emerging from) the jth mirror, which is positive (or negative) for the ray of light propagating toward the positive (or negative) z axis. Thus we have $n_1 = -n_2 = n_3$, and $n_j' = -n_j$ for the incident and reflected rays on the jth mirror. R_j is the radius of curvature of the jth mirror, which is positive (or negative) when the center of curvature is on the right-hand (or left-hand) side of the surface. m is the diffracted order and the grating spacing is assumed to be of $p_2 = p$, while $p_1 = p_3 = \infty$. If we let $b_j = n_j \pi w_j^2/\lambda$ (or $b_j' = n_j' \pi w_j'^2/\lambda$) in terms of the minimum spot radius w_j (or w_j') for the beam incident on (or emerging from) the jth mirror, $(0, \bar{y}_j, \bar{z}_j)$ (or $(0, \bar{y}_j', \bar{z}_j')$) can be taken equal to the local coordinates of the center of the Gaussian beam incident on (or emerging from) the jth mirror. $\bar{u}_j$ (or $\bar{u}_j'$) becomes the slope angle of the Gaussian beam incident on (or emerging from) the jth mirror, which is positive (or negative) when measured clockwise (or anti-clockwise) from the beam axis to the z axis. $\bar{y}_{P_j}$ is the height at which the beam axis meets the tangent plane of the jth mirror. The paraxial parameters for the jth and (j+1)th mirrors are linked by the distance d_j' between the jth and (j+1)th mirrors. In a concentric-type mirror system under consideration,

$$\begin{aligned}
&d_j' = R_j - R_{j+1},\\
&\bar{z}_{j+1} = \bar{z}_j' - d_j', \qquad \bar{y}_{j+1} = \bar{y}_j',\\
&\bar{y}_{P_{j+1}} = \bar{y}_{P_j} + d_j'\bar{u}_j',
\end{aligned} \tag{5}$$

for j = 1 and 2. In addition, we have to choose

$$\bar{u}_1 \simeq \left(\frac{2}{R_1} - \frac{1}{R_2} \right) \bar{y}_1, \tag{6}$$

so that the beam axis can pass through the center of the aperture stop (i.e., $\bar{y}_{P_2} = 0$).

In cases where the terms of up to fourth order in the aperture variables are taken into account, the wave function of the light converging to O_3' can be derived in a similar manner as in Ref. [7]. The wave function at the local coordinates $(x_3', y_3', \bar{z}_3')$, referenced to the third concave mirror, is given by