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Optimization of wavefront coding imaging system based on the phase transfer function

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ABSTRACT

The variation of the phase transfer function with respect to defocus causes linear image shift and image artifacts on the restored images in wavefront coding imaging system with asymmetrical phase masks. To reduce the effect of the phase transfer function on the restored image according to the signal to noise ratio, in this paper, an optimization function based on the phase transfer function is proposed. By using this function with the signal to noise ratio equal to 30 dB, optimization and simulation result for the cubic phase mask and the tangent phase mask are presented. The simulation result shows that the tangent phase mask produces more benefit to reduce image artifacts on the restored image.

1. Introduction

Wavefront coding imaging system is the combination between the optical imaging step and the digital processing step. By placing a phase mask in the pupil plane of wavefront coding imaging system, the optical transfer function (OTF) or the point spread function (PSF) is less sensitive over a large range of defocus. The middle images are the same blurred images. In order to obtain sharp images, we should apply the digital processing step using deconvolution kernel to blurred intermediate images. The important part to wavefront coding imaging system lies in the design of suitable phase masks to obtain defocus invariant imaging characteristic. In the past twenty years, many phase masks to increase the depth of field have been introduced, such as the cubic phase mask [1], the logarithmic phase mask [2], the exponential phase mask [3], the sinusoidal phase mask [4], the polynomial phase mask [5], the tangent phase mask [6], the free-form phase mask [7], the rational phase mask [8], and the high-order phase mask [9].

Generally speaking, in order to analyze and evaluate imaging performance of wavefront coding imaging with a phase mask, mask parameters of the phase mask should be optimized firstly because the optimal value of mask parameters enables to produce the effective imaging performance of wavefront coding imaging system with this phase mask. Based on the modulation transfer function (MTF), the PSF and the OTF, many optimization functions to choose the optimal value of mask parameters have been suggested in [10–17]. However, optimization function based on the phase transfer function (PTF) has been yet used to choose optimal value of mask parameters. Moreover, an analysis about the PTF for wavefront coding imaging system with the cubic phase mask was shown in [18], and a net modulation of phase transfer function includes two parts: a linear part to the spatial frequencies causes a linear image shift in the restored image and the other non-linearly part to the spatial frequencies causes image artifacts on the restored image. Additionally, the digital processing not only restores sharp image, but also amplifies the underlying; therefore, the noise amount will increase. The effect of noise amount on the restored image depends on signal to noise ratio (SNR). Based on the phase transfer function with SNR, in this paper, we proposed an optimization function which can be used to obtain optimal mask parameters of asymmetrical phase masks.

The paper is organized as follows. Section 2 presents the proposed evaluation function based on the phase transfer function. Optimization and simulation results for the cubic phase mask and the tangent phase mask are shown in Section 3. Finally, the conclusions are presented in Section 4.

2. Evaluation function based on the phase transfer function

The PTF of the optical transfer function for a given defocus ψ in wavefront coding imaging system can be given by,

$$\phi(u, v; \psi) = \arctan\left\{\frac{image\,[H(u, v; \psi)]}{real\,[H(u, v; \psi)]}\right\} \quad (1)$$

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where *image*, *real* are the imaginary, real parts of the optical transfer function, respectively; H is the optical transfer function; both u and v are the spatial frequencies; ψ is the defocus parameter given by,

$$\psi = \frac{\pi L^2}{4\lambda}\left(\frac{1}{f} - \frac{1}{d} - \frac{1}{d_0}\right) \tag{2}$$

where L is the pupil plane dimension; λ is the wave length of light; f, d, d_0 are the focal length, the object distance, and the image distance, respectively.

The OTF at a given defocus ψ is equal to Fourier transform of the point spread function and can be presented by,

$$H(u, v; \psi) = FFT\ [PSF(x_0, y_0; \psi)] \tag{3}$$

where x_0, y_0 are the spatial positions.

The PSF of wavefront coding imaging system with the generalized pupil function, $P(x, y)$, where x and y are the coordinates in the pupil plane, can be presented by,

$$PSF(x_0, y_0; \psi) = |FFT[P(x, y)]|^2 \tag{4}$$

The generalized pupil function in wavefront coding imaging system with a phase mask $f(x, y)$ and a defocus parameter ψ can be presented by,

$$P(x, y) = \begin{cases} \frac{1}{\sqrt{2}}\exp[if\ \ (x, y) + i\psi(x^2 + y^2)] & \text{if}\ \ |x| \le 1, |y| \le 1 \\ 0 & \text{other} \end{cases} \tag{5}$$

In the absence of noise, the restored image by using the Wiener filter can be given by,

$$I(x_0, y_0; \psi) = iFFT^{-1}[F(u, v)H(u, v; \psi)O(u, v)] \tag{6}$$

where $iFFT^{-1}$ is the inverse Fourier transform. O is the Fourier transform of the object o. F is the Wiener filter. When the Wiener filter is driven from the in-focus OTF [1,7,8], the quality degradation of the restored image depends on the variation of the MTF and the PTF to defocus. In other case, when the Wiener filter is driven from the defocus matching OTF [11,12,15,19], the restored image is identical to the diffraction limited image of a traditional imaging system. Therefore, it does not depend on the variation of the MTF and the PTF to defocus. To consider only the effect of the variation of the PTF to defocus in the restored image, we should eliminate the effect of the variation of the MTF to defocus in the restored image. To resolve this problem, we use the Wiener filter driven from the MTF of the defocus matching OTF and the PTF of the in-focus OTF and it can be presented by,

$$F(u, v) = \frac{\exp(-i\phi(u, v; \psi = 0))}{MTF(u, v; \psi)} H_{dl}(u, v) \tag{7}$$

where H_{dl} is the diffraction-limited in-focus OTF of a traditional imaging system; $MTF(u,v; \psi)$ is the modulation transfer function of wavefront coding imaging system.

By inserting Eq. (7) into Eq. (6), the restored image can be rewritten as,

$$\begin{aligned} I(x_0, y_0; \psi) = iFFT^{-1}[&\exp(i\phi(u, v; \psi) \\ &- i\phi(u, v; \psi = 0))H_{dl}(u, v)O(u, v)] \end{aligned} \tag{8}$$

To setting,

$$PTF(u, v; \psi) = \phi(u, v; \psi) - \phi(u, v; \psi = 0) \tag{9}$$

By inserting Eq. (9) into Eq. (8), the restored image can be presented by,

$$I(x_0, y_0; \psi) = iFFT^{-1}[\exp(iPTF(u, v; \psi))H_{dl}(u, v)O(u, v)] \tag{10}$$

Fig. 1. LENA target.

As can be seen from Eq. (10), the restored image, which only depends on the variation of the PTF to defocus, does not depend on the variation of the MTF to defocus.

In the presence of noise, the digital processing by using the Wiener filter not only restores sharp final image, but also amplifies the underlying noise. Consequently, in the presence of noise, the restored final image can be given by,

$$I_{\mathrm{Final}}(x_0, y_0; \psi) = I(x_0, y_0; \psi) + n_{\mathrm{noise\text{-}amplification}}(x_0, y_0) \tag{11}$$

where $n_{\mathrm{noise\text{-}amplification}}$ is the noise amplification due to the digital processing.

As shown in [10,11], the mean square error (MSE) between the restored final image for a mask parameter a and a defocus parameter b_k, *where* $k = 1, 2, \ldots, m$, and the object, can be presented by,

$$\Delta_k = \left\langle \left| I_{\mathrm{final}}(x_0, y_0; \psi) - o(x_0, y_0) \right|^2 \right\rangle \tag{12}$$

where $\langle\rangle$ is the average value of all the spatial positions. A sum of MSEs for all values of defocus can be presented by,

$$Metric = \Delta_1 + \Delta_2 + \ldots + \Delta_m \tag{13}$$

The aim of the work is to find a mask parameter a that produces the minimum value of *Metric* as given in Eq. (13).

3. Simulation result

In order to perform numerical simulations with the optimization function as presented in Eq. (13), we here use LENA target as shown in Fig. 1, a range of defocus from 0 to 20, sign noise ratio equal to SNR = 30 dB, and m = 11.

Firstly, we consider optimization of the most common asymmetrical cubic phase mask for imaging system less sensitive to

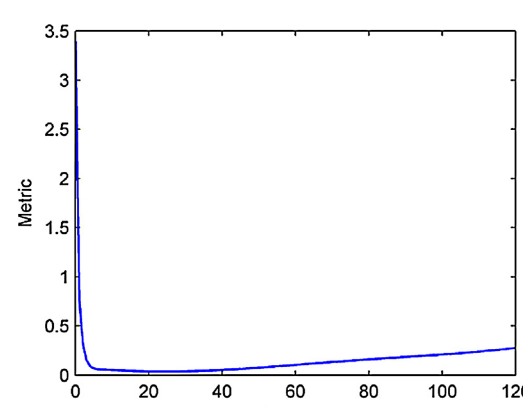

Fig. 2. Metric curve for the cubic phase mask.