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Theoretical investigations of the performance of the active laser region by the calculation of the optical gain

R. Ben Dhafer [a], A. Ben Ahmed [a,*], H. Saidi [a], S. Ridene [a,b], H. Bouchriha [a]

[a] Laboratoire Matériaux Avancés et Phénomènes Quantiques, Faculté des Sciences de Tunis, Université de Tunis El Manar, Campus Universitaire, 2092 Tunis, Tunisia
[b] Département de physique, Faculté des Sciences de Bizerte, université de Carthage, 7021 Zarzouna, Bizerte, Tunisia

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ABSTRACT

In this work, we discuss the performance of the active laser region realized respectively by bulk (3D) and quantum well (2D) semiconductors, with emphasis on the basic behavior of the optical gain. Calculations are based on a semi-classic model used to describe the performance of the bulk semiconductor (3D) and quantum well (2D) actives zones. It is revealed that the use of quantum well structures results in improvement of these properties and brings several new concepts to the active laser region.

1. Introduction

In the past few years, the quantum well laser structure emitting at mid-infrared (2–5 μm) presented an important interest for both theoretical [1–4] and experimental [5,6] considerations. This is due to the fact that these laser structures have been extensively used in spectroscopy, environmental monitoring, medical diagnostics, and military countermeasure systems. An interesting question is related to the active laser region which can be fabricated with bulk semiconductors or quantum well. Since the emission wavelength of a semiconductor corresponds to its band-gap energy, research focuses on engineering new materials which have their band gaps at custom-designed energies [7–9]. In the early years of semiconductor optoelectronics, the band gaps that could be achieved were largely determined by the physical properties of key III–V materials such as GaAs and its alloys such as AlGaAs and InGaAs [10,11]. Then in 1970 a major breakthrough occurred when Esaki and Tsu invented the semiconductor quantum well and superlattice [12]. They realized that quantum confinement would be of interest to engineer electronic states with custom-designed properties. In this work, semi-classical model incorporating Maxwell-equations and matrix density is used to study the performances of bulk and quantum well semiconductors in term of optical gain. The effects of temperature and quantum well width are analyzed theoretically. The numerical results clearly show that decreasing of temperature and well widths, the optical gain increases. The layout of this paper is as follow: The theoretical details to calculate optical gain for bulk and quantum wells (QWs) semiconductors are given in Section 2. Results and discussions on the optical gain performances for some bulk materials such as InAs, GaSb and GaAs and for InAsN/GaSb QW laser structure are presented in Section 3. Finally, a summary is given in Section 4.

2. Theoretical considerations

The matrix density approach is considered necessary for the calculation of the optical gain by taking into account of statistical density rate. By using the Dirac notation, the operator of the matrix density is given as follows:

$$\rho = \sum_n f(E_n) \left|\Psi_n\right\rangle \left\langle\Psi_n\right| \quad (1)$$

where $f(E_n)$ is the occupation probability associated to the electron state $\left|\Psi_n\right\rangle$.

The fundamental equation that governs the evolution in time of ρ is the Liouville equation:

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H, \rho] \quad (2)$$

where $H = H_0 + H_d$ represents the total Hamiltonian, H_0 and $H_d = -\vec{p}_d \cdot \vec{E}$ represent respectively the non-perturbative and perturbative Hamiltonians, $p_d = e\,r$ is the momentum operator of the electric

* Corresponding author.
E-mail address: amira.ben.hmed@gmail.com (A. Ben Ahmed).

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dipole associated to the particle, e is the elementary charge. For the conduction band (CB), the matrix density can be written as:

$$\rho_{cc} = f(E^c - E_F^c) \tag{3}$$

As well as, for the valence band (VB), we have:

$$\rho_{vv} = f(E^v - E_F^v) \tag{4}$$

where f is the Fermi–Dirac occupation rate, E_F^c and E_F^v are respectively the quasi-Fermi levels for the CB and VB. The general evolution equation of the matrix density in the electric-dipole approximation is given by [13]:

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H_0 - p_d \cdot E, \rho] - \frac{1}{2}\left[\Gamma\left(\rho - \rho^{(0)}\right) + \left(\rho - \rho^{(0)}\right)\Gamma\right] \tag{5}$$

where $H_0 = \begin{bmatrix} E^c & 0 \\ 0 & E^v \end{bmatrix}$, $\Gamma = \begin{bmatrix} \gamma_c & 0 \\ 0 & \gamma_v \end{bmatrix}$ is the operator describing the intraband relaxation time, and $p_d \cdot E = \begin{bmatrix} 0 & R_{cv}E \\ R_{vc}E & 0 \end{bmatrix}$ where $R_{cv} = \langle \Psi_c | e \cdot r | \Psi_v \rangle$ and E is the electric field. We replace these terms in Eq. (5), we obtain the following expression:

$$\frac{\partial \delta\rho_{cv}}{\partial t} = \frac{1}{i\hbar}(E^c - E^v - i\hbar\gamma_{\text{int}})\partial\rho_{cv} - \frac{R_{cv}}{i\hbar}E\left[f\left(E^c - E_F^c\right) - f\left(E_F^v - E^v\right)\right] \tag{6}$$

We assume that the operator $\delta\rho_{cv}$ has the same evolution time as the excitatory field:

$$\delta\rho_{cv} = \delta\tilde{\rho}(\omega)e^{-i\omega t} + \delta\tilde{\rho}(-\omega)e^{i\omega t} \tag{7}$$

where $\delta\tilde{\rho}(\omega) = -\frac{R_{cv}\tilde{E}\left[f\left(E^c - E_F^c\right) - f\left(E_F^v - E^v\right)\right]}{(E^c - E^v - \hbar\omega + i\hbar\gamma_{\text{int}})}$ and $\delta\tilde{\rho}(-\omega) = -\frac{R_{cv}\tilde{E}^*\left[f\left(E^c - E_F^c\right) - f\left(E_F^v - Eh\nu\right)\right]}{(E^c - E^v + \hbar\omega + i\hbar\gamma_{\text{int}})}$, $\gamma_{\text{int}} = \frac{1}{\tau_{\text{int}}} = \frac{\gamma_c + \gamma_v}{2}$ is the inverse intra-band relaxation time.

The complex electric susceptibility $\tilde{\chi}_e$ can be deduced from the average electric polarization per unit volume denoted by $\langle P \rangle = \frac{1}{V}Tr(\rho_{cv}R_{cv}) = \varepsilon_0\tilde{\chi}_e(\omega)\tilde{E}e^{-i\omega t} + \varepsilon_0\tilde{\chi}_e(-\omega)\tilde{E}^*e^{i\omega t}$. Thereby, the complex electric susceptibility will be written as $\tilde{\chi}_e(\omega) = \chi'(\omega) + i\chi''(\omega) = \frac{1}{V}\frac{|R_{cv}|^2}{\varepsilon_0}\frac{\left[f\left(E^c - E_F^c\right) - f\left(E_F^v - E^v\right)\right]}{(\hbar\omega - E^c - E^v - i\hbar\gamma_{\text{int}})}$ where V is the volume of the active zone. This equation can be generalized for the inter-band transitions involving several states of the CB and VB:

$$\tilde{\chi}_e(\omega) = \frac{1}{V\varepsilon_0}\sum_{cv}\left|R_{cv}\right|^2\frac{\left[f\left(E^c - E_F^c\right) - f\left(E_F^v - E^v\right)\right]}{(\hbar\omega - E^c - E^v - i\hbar\gamma_{\text{int}})} \tag{8}$$

From the complex electric susceptibility, it is possible to define several fundamental quantities such as the optical gain $G(\omega)$:

$$G(\omega) = \frac{e^2}{m_0^2 n_r c\varepsilon_0\omega}\frac{1}{V}\sum_{cv}\left|M_{cv}\right|^2\left[f\left(E^c - E_F^c\right) - f\left(E_F^v - E^v\right)\right] \times \frac{\hbar\gamma_{\text{int}}}{(\hbar\omega - E^c - E^v)^2 + (\hbar\gamma_{\text{int}})^2} \tag{9}$$

2.1. Optical gain for bulk semiconductors

Taking into account the density of state, the optical gain for the bulk semiconductors can be expressed as follows:

$$G(\hbar\omega) = \frac{e^2}{2m_0\omega nc}\sum_{c,v}\left|M_{cv}\right|^2\left(\frac{2m_r^*}{\hbar^2}\right)^{3/2}\sqrt{\hbar\omega - E_g}(f_v - f_c) \tag{10}$$

where $\left|M_{cv}\right|^2 = \frac{E_p}{4}$ [2], m_r^* is the reduced effective mass expressed as $m_r^* = \frac{m_c \times m_v}{m_c + m_v}$, where m_c and m_v are respectively the conduction and valence effective masses. The computation of the optical gain requires firstly the calculation of quasi-Fermi levels obeying on the electrical neutrality condition. From the carrier density of the conduction and valence band, we have:

$$N_{3D} = 2\left(\frac{m_c k_B T}{2\pi\hbar^2}\right)^{3/2}\exp\left(\frac{E_F^c - E_g}{k_B T}\right) \tag{11}$$

$$P_{3D} = 2\left(\frac{m_v k_B T}{2\pi\hbar^2}\right)^{3/2}\exp\left(\frac{-E_F^v}{k_B T}\right) \tag{12}$$

Then, we obtain the following expressions for the quasi-Fermi energies:

$$E_F^c = E_g + \frac{1}{\beta}\log\left(\frac{N_{3D}}{n_c}\right) \tag{13}$$

$$E_F^v = -\frac{1}{\beta}\log\left(\frac{P_{3D}}{p_c}\right) \tag{14}$$

2.2. Optical gain for quantum well

In two dimensional semiconductor, i.e. quantum well, it is well known that the optical gain can be obtained with the contributions of all the allowed transitions between electron and hole subbands according to the density matrix theory and taking into account the density of states ρ_{DOS}^{2D} [3]:

$$G(\omega) = \frac{e^2}{m_0^2 n_r c\varepsilon_0\omega}\frac{1}{L_p}\sum_{nm}J_{cv}^{nm}\int_0^{+\infty}\rho_{DOS}^{2D}\left|M_{cv}\left(k_\rho\right)\right|^2 \times \left[f_n^c\left(E_{n,k_\rho}^c - E_F^c\right) - f_m^v\left(E_F^v - E_{m,k_\rho}^v\right)\right] \times \frac{\hbar/\tau_{\text{int}}}{\left(\hbar\omega - (E_{n,k_\rho}^c - E_{m,k_\rho}^v) - E\right)^2 + \left(\hbar/\tau_{\text{int}}\right)^2}dE \tag{15}$$

with $\left|M_{cv}(k_\rho)\right|^2 = \delta(m_0/6)E_p$, where $\delta = 3/2$ for TE polarization. c and ε_0 are the velocity of light and permittivity of free space; n_r, L_p, J_{cv}^{nm} and $\rho_{DOS}^{2D} = \frac{m^*}{\pi\hbar^2}\sum_i\sigma(E - E_i)$ are the refractive index, the effective width of the active zone, the wave-function overlap and the two-dimensional density of states, respectively. f_n^c and f_m^v are Fermi functions for the nth subband in the conduction band and mth subband in the valence band, respectively as well as the energy dispersion curves E_n^c and E_m^v, and τ_{int} is the intraband relaxation time. We assumed that τ_{int} is a constant value, $\tau_{\text{int}} = 1 \times 10^{-14}$ s [14]. Note that, the numerical calculation of quasi-Fermi levels must consider the population of carriers in confined conditions (2D states) and the neutrality condition attributed in the active region, we obtain:

$$N_{2D} = \frac{m}{\pi\hbar^2}k_B T\log(1 + \exp -\beta(E_c - E_F^c)) \tag{16}$$

$$P_{2D} = \frac{m}{\pi\hbar^2}k_B T\log(1 + \exp -\beta(E_F^v - E_v)) \tag{17}$$

Therefore, we have:

$$E_F^c = E_c + k_B T\log\left(\exp\left(\frac{N_{2D}}{n_c}\right) - 1\right) \tag{18}$$

$$E_F^v = E_v - k_B T\log\left(\exp\left(\frac{P_{2D}}{p_c}\right) - 1\right) \tag{19}$$

3. Results and discussion

In order to investigate the performances for some bulk materials (3D), we have taken the case of GaAs, InAs and GaSb semiconductors which are widely used in the optoelectronic devices. In all the