Optik 127 (2016) 1237–1244

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Short Note

# Beam propagation and mode coupling study in a coupled waveguide structure by using scalar finite element method

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ARTICLE INFO

*Article history:*
Received 21 February 2015
Accepted 30 October 2015

*PACS:*
42.79.Gn
42.81.–I
42.82.–m

*Keywords:*
Computation method
Coupled waveguide structure
Waveguide dispersion
Mode propagation
Transparent boundary condition

ABSTRACT

Here we present the propagation of pulse in a coupled waveguide structures. Finite element method has been effectively applied to analyze the closely coupled waveguide structure. The mode coupling phenomena is well described in this paper. It is demonstrated that there are substantial amount of power exchange takes place between various modes. We excite the structure with original mode pattern which can be well approximated by slightly deviated Gaussian shape pulse. Then we apply the mode propagation concept to estimate the mode beating phenomena in coupled structure. The effect of waveguide separation is well demonstrated by using examples. We too derive the Fresnel equation and discuss a solution strategy in terms of the finite element method. We apply the transparent boundary condition to avoid the unwanted reflections from the artificial computation windows.



## 1. Introduction

Coupled waveguides have been the object of several investigations in recent years for the promising prospect of designing of optical switches and optical coupler to mention few [1–5]. Coupling between optical modes is compulsory in the design of integrated optical devices. Coupled mode theory describes this energy exchange and serves as the primary tool for designing of optical couplers, switches and filters. In this paper we have explored some applications of coupling between three closely located waveguides based on finite element method approach. We found that coupling is maximized when the propagation coefficient, $\beta$ for the various mode's are equal. We can enhance the coupling between dissimilar waveguides by using phase matching technique. The coupling of radiation from a waveguide into free space is an important problem in optoelectronics. The calculation is difficult because the guided modes couple to free space modes, which are not normalizable. The design of such couplers is as much art as engineering. Finally we have applied our approach to analyze the actual lowest order mode propagation to this structure. The coupling period of the power is verified by Beam Propagation method [6–23].

## 2. Finite element method analyses for coupled waveguide structure

In this section, the brief formulation of FEM is described while considering both TE and TM mode as an example in the coupled waveguide structure [3,4]. A planar waveguide is characterized by a permittivity profile which depends on one coordinate only, say $x$. If $\varepsilon = \varepsilon(x)$ is a varying function with $x$-coordinates we speak of an arbitrary index waveguide. A slab waveguide consists of step index permittivity profile. The lowest is the substrate, on top of which there are one or many thin films, the topmost region is the cover. A coupled waveguide is characterized by a piecewise constant permittivity profile as shown in Fig. 1, the transverse region, $4 \le x \le 12$, which has a three coupled waveguide structure is denoted as the core and the refractive indices in the cover and the substrate are assumed to be constant. The waveguide separation for this closely coupled waveguide structure is 1 μm. The maximum refractive index in the core is $n_1$, and the refractive indices in the cover and the substrate are $n_0$ and $n_s$ respectively. Here we assume $n_s \ge n_0$. The wave equation for the TE mode is given by [3–8];

$$\frac{d^2E_y}{dx^2} + \{k^2n(x)^2 - \beta^2\}E_y = 0, \tag{1}$$

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http://dx.doi.org/10.1016/j.ijleo.2015.10.234
0030-4026/

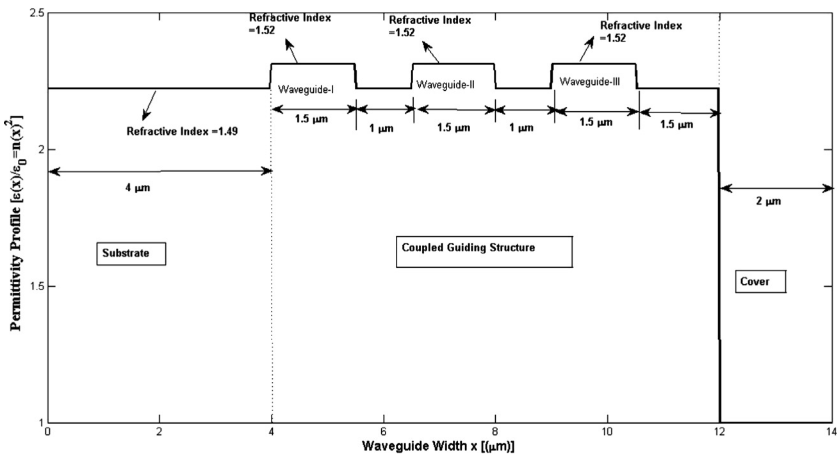


**Fig. 1.** Permittivity profile $\varepsilon(x)$ versus cross section coordinate $x$ for closely coupled waveguides.

and for the TM mode [19];

$$\frac{d}{dx}\left(\frac{1}{n^2}\frac{dH_y}{dx}\right)+\left(k^2-\frac{\beta^2}{n}\right)H_y=0, \tag{2}$$

The boundary conditions require the continuity of $E_y$ for TE mode is,

$$H_z=\frac{j}{\omega^{\mu_0}}\frac{dE_y}{dx} \tag{3}$$

and the boundary conditions require the continuity of $H_y$ for TM mode is,

$$E_z=-\frac{j}{\omega\varepsilon_0 n^2}\frac{dH_y}{dx} \tag{4}$$

at various interfaces. Before transforming the wave Eq. (1) and boundary conditions (Eqs. (3) and (4)) into the variational problem, the parameters are normalized as

$$\rho=\frac{x}{a},\quad E_y(x)=R(\rho),\quad D=\frac{A}{a}. \tag{5}$$

Here "A" is the boundary at various coupled waveguide interface. The wave equation and boundary condition are then rewritten for TE mode a

$$\frac{d^2R}{d\rho^2}+[v^2q(\rho)-w^2]R=0. \tag{6}$$

$$R(\rho)\ \text{ and }\ \frac{dR(\rho)}{d\rho}\ \text{ are continuous at }\ \rho=0\ \text{ and }\ \rho=D. \tag{7}$$

Here, normalized transverse wave-number $w$, normalized frequency $v$, and normalized refractive-index distribution $q(\rho)$ are defined by

$$\left.\begin{aligned} w&=a\sqrt{\beta^2-k^2n_s^2},\\ v&=ka\sqrt{n_1^2-n_s^2},\\ q(\rho)&=\frac{n(x)^2-n_s^2}{n_1^2-n_s^2}\end{aligned}\right\} \tag{8}$$

where $k$ is the wave number. The solution of the wave Eq. (6) under the constraints of the boundary condition Eq. (7) is obtained as the solution of the variational problem that satisfies the stationary condition of the functional.

$$I[R]=-\int_{-\infty}^{\infty}\left(\frac{dR}{d\rho}\right)^2d\rho+\int_{-\infty}^{\infty}[v^2q(\rho)-w^2]R^2d\rho \tag{9}$$

the eigenvalue matrix elements for the TE and TM modes are expressed as,

$$\left.\begin{aligned} c_{0.0}&=\eta_0-(3q_0+q_1)\eta_0\frac{\upsilon^2}{12}\delta^2+\eta_0\frac{w^2}{3}\delta^2+\eta_0w_0\delta,\\ c_{i,i}&=(\eta_{i-1}+\eta_i)-(q_{i-1}\eta_{i-1}+3q_i\eta_{i-1}+3q_i\eta_i+q_{i+1}\eta_i)\frac{\upsilon^2}{12}\delta^2+(\eta_{i-1}+\eta_i)\frac{w^2}{3}\delta^2\quad(i=1-(N-1)),\\ c_{i,i+1}&=c_{i+1,i}=-\eta_i-(q_i+q_{i+1})\,\eta_i\frac{\upsilon^2}{12}\delta^2+\eta_i\frac{w^2}{6}\delta^2\quad(i=0-(N-1)),\\ c_{N,N}&=\eta_{N-1}-(q_{N-1}+3q_N)\eta_{N-1}\frac{\upsilon^2}{12}\delta^2+\eta_{N-1}\frac{w^2}{3}\delta^2+w\delta,\end{aligned}\right\} \tag{10}$$

where

$$n_i=\begin{cases}1 & \text{TE mode}\\ \dfrac{n_s^2}{n^2(\rho_i)} & \text{TE mode}\end{cases} \tag{11}$$

$$n_{s0}=\begin{cases}1 & \text{TE mode}\\ \dfrac{n_s^2}{n_0^2} & \text{TE mode}\end{cases} \tag{12}$$

For nontrival solutions expect for $R_0=R_1=\cdots=R_N=0$, the determinant of the matrix $C$ should be

$$\det(C)=0 \tag{13}$$

where the element of the matrix C is given by Eq. (10) and discretization step $\delta$ is given by $\delta=D/N$. Eq. (13) is a dispersion equation (eigenvalue equation) for the TE/TM modes in coupled waveguide