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Relativistic nonlinear propagation of rippled Gaussian laser beam in plasma

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ABSTRACT

This paper presents an analysis of the relativistic nonlinear propagation of a rippled Gaussian laser beam in plasma. Considering the nonlinearity as arising owing to relativistic variation of mass, and following the WKB and paraxial-ray approximations, the phenomenon of self-focusing of rippled laser beams is studied for arbitrary magnitude of nonlinearity. It is seen that small ripple on the axis of the main beam grows very rapidly with distance of propagation as compared with the self-focusing of the main beam. Based on this analogy, we have analyzed relativistic self-focusing of rippled beams in plasmas. The relativistic intensities with saturation effects of nonlinearity allow the nonlinear refractive index in the paraxial regime to have a slower radial dependence, and thus the ripple extracts relatively less energy from its neighborhood.

1. Introduction

New short-pulse laser technology has recently made possible the production of extremely intense laser sources at multiterawatt level [1,2]. The focused intensities obtained are very high, of the order of 10^{18} W/cm^2, and further developments are aimed at intensities exceeding 10^{20} W/cm^2. The development of such high-intensity lasers has lead to the possibility of observing relativistic effects when a laser pulse interacts with a fully ionized plasma. The propagation of a high-intensity laser pulse through a fully ionized plasma is a basic physics problem, and is of great interest with regard to practical applications for compact X-ray lasers [3], laser-plasma-based particle accelerators [4] and the fast ignitor scheme for studies of inertial confinement fusion (ICF) [5]. The relativistic filamentation instability can lead to modification of the propagating pulse by spatially modulating the laser intensity transverse to the direction of propagation. The relativistic filamentation and self-focusing instabilities have been studied for a number of years, and are important because one needs to understand laser propagation at high intensities before one can interpret the results from other nonlinear phenomena [6,7].

In an experimental situation where intense laser or electromagnetic beams traveling through nonlinear self-focusing media result in multiple filament formation, there is a one-to-one correspondence between filaments and intensity spikes riding with the incident laser beam, as studied earlier by Abbi et al. [8]. The origin of the filamentation instability may be attributed to small-scale density perturbations (resulting from quasineutrality) or small-scale intensity spikes associated with the main beam. The perturbation grows at the cost of the main beam, and this is detrimental to laser-induced fusion. In laser plasma experiments, the filamentary structure created in an underdense plasma undergoes self-focusing. The self-focused filaments spoil the symmetry of the energy deposition as well as triggering parametric instabilities that may lead to back- and side-scattering of the laser beam. Thus direct and indirect experimental evidence reveals that an apparently smooth laser beam has intensity spikes that may lead to distortion of self-focusing in nonlinear media [9,10].

Filamentation instability (hot spot formation) of laser beams in plasmas has been studied in detail in the past, and it is an area of continued interest. These studies are also relevant to laser electron acceleration and laser driven fusion. The relativistic filamentation instability can lead to modification of the propagation pulse by spatially modulating the laser intensity transverse to the direction of propagation.

Currently there is significant interest in intense short pulse laser propagation through plasmas with density ripple [11]. Some of

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those studies have been motivated by the need for resonant generation of third and higher harmonics. In a homogeneous plasma, the harmonic generation process is a non-resonant one as the nth harmonic wave number $k_n \neq nk_1$, where k_1 is the wave number of the fundamental laser. A density ripple of suitable wave number $q = (k_n - nk_1)$, may turn the process into a resonant one. Hence, efforts have been made to create suitable density ripples in plasmas. This kind of density ripple may be observed in gas-jet plasma experiment. Lin et al. [12] have recently used a 10 TW, 45 fs, 810 nm and 10 Hz Ti:Sapphire laser, to accomplish this. The laser beam was spilt into two beams. One beam, moving along x was a machining pulse for producing the periodic plasma density profile and the other one, launched 10 ns later and propagating along z was a probe pulse. The machining pulse, having periodic intensity variation in transverse direction z, and intensity at the maxima exceeding the threshold for optical field ionization, was projected transversely on a neutral gas jet. In positions of intensity, maxima plasma was formed. After the passage of the pulse, the interlacing layers of high density neutral gas low density plasma were formed. When a large amplitude probe pulse propagates along z through such structure, the front of the pulse ionizes the gas, and its subsequent portion encounters plasma with longitudinal density ripple.

Chessa et al. [13] have reported ring formation over a high power Gaussian laser beam as it propagates through a tunnel ionizing gas. Leemans et al. [14] have seen ring formation in their experiment. Liu and Tripathi [15] have interpreted some of these results as a consequence of stronger self-defocusing of paraxial rays as compared to marginal rays. This mechanism does not predict ring formation when nonlinear refraction causes self-focusing. However, media with self-focusing nonlinearity are known to be susceptible to filamentation instability, hence, a ring perturbation over a Gaussian beam may also grow to a large level in the course of propagation. This is due to the fact that the ring region, with higher intensity, would have higher index of refraction and would attract higher energy from the neighborhood and grow.

A lot of theoretical and experimental work has been reported related to beat wave excitation (BWE), by two laser beams and laser induced parametric instabilities in plasmas like stimulated Raman scattering (SRS), and stimulated Brillioun scattering (SBS) by a single laser beam. But these studies used spatially smooth laser beam, which is not realistic. The direct and indirect experimental evidence [16] revels that the smooth looking laser beams have strong intensity ripples (spikes), which may lead to the formation of filaments (hot spots) in plasmas. Therefore, the parametric interaction and plasma beat wave excitation (PBWE) can be considered coherently only inside a spike. But the experimental observations, which generally reflect integrated quantity, results from an average over a statistical distribution of independent ripples with different intensities. One can use different models of a laser beam having single hot spot [17], two closely hot spots [18], and a randomized laser beam having ripples of different intensities [19].

In view of the ongoing development of ultra-intense short-pulse lasers, we present here an analysis of the relativistic self-focusing of a Gaussian laser beam with a ring ripple superimposed on it for arbitrary large nonlinearity. In Section 2, the general equations for the self-focusing are presented with equation governing the variation of the beam width parameter with distance of propagation. In Section 3, an expression for the nonlinearity is given, and the self-trapping condition and the critical power are presented. Results and a discussion are given in Section 4.

2. Relativistic self-focusing equation of rippled laser beam

Consider the propagation of a Gaussian laser beam with a ring ripple superimposed on it in a homogeneous collisionless plasma along the z-direction. The electric field at the fixed plane $z = 0$ of the main beam may be represented by

$$\mathbf{E}_0|_{z=0} = \mathbf{E}_{00} \exp\left(-\frac{r^2}{2r_0^2}\right) \exp(i\omega t) \tag{1}$$

where ω is the angular frequency of the laser beam, r is the radial coordinate of the cylindrical coordinate system and r_0 is the initial width of the main beam. The electric field of the ring superimposed on the main beam may be expressed as

$$\mathbf{E}_1|_{z=0} = \mathbf{E}_{10}\left(\frac{r}{r_{10}}\right) \exp\left(-\frac{r^2}{2r_{10}^2}\right) \exp(i\omega t - \phi_p) \tag{2}$$

with ϕ_p is the phase difference in between the main beam and the ripple; r_{10} is the width of the ripple; the maximum field of the ripple at $r = r_{10}$. The total electric vector of the beam can thus be written as $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$. The intensity distribution of the rippled Gaussian laser beam is thus given by

$$\begin{aligned}\mathbf{E}.\mathbf{E}^*|_{z=0} &= E_{00}^2 \exp\left(-\frac{r^2}{r_0^2}\right) \\ &\times \left\{1 + 2\frac{E_{10}}{E_{00}}\frac{r}{r_{10}} \cos\phi_p \exp\left[\frac{r^2}{2}\left(\frac{1}{r_0^2} - \frac{1}{r_{10}^2}\right)\right]\right. \\ &\left. + \frac{E_{10}^2}{E_{00}^2}\left(\frac{r}{r_{10}}\right)^2 \exp\left[r^2\left(\frac{1}{r_0^2} - \frac{1}{r_{10}^2}\right)\right]\right\}\end{aligned} \tag{3}$$

The wave equation governing the electric vector of the beam in plasmas with the dielectric constant can be written as

$$\nabla^2\mathbf{E} + \frac{\omega^2}{c^2}\varepsilon\mathbf{E} = 0 \tag{4}$$

Keeping in mind that $(c^2/\omega^2)|\nabla \ln\varepsilon| \ll 1$, within the WKB (for Wentzel, Kramers, and Brillouin) approximation, we neglect the term $\nabla(\nabla\cdot\mathbf{E})$ while writing Eq. (4). The nonlinear dielectric constant of the medium for arbitrary large nonlinearity is [20]

$$\varepsilon\left(\left\langle\mathbf{E}.\mathbf{E}\right\rangle\right) = \varepsilon_0 + \phi\left(\left\langle\mathbf{E}.\mathbf{E}\right\rangle\right) \tag{5}$$

In the paraxial-ray approximation, one generally expands ϕ around $\phi \approx 0$. However, with such an expansion, one can study only those cases where $\phi \ll \varepsilon_0$. To study self-focusing for arbitrary large ϕ, the nonlinear dielectric constant of the medium at $r = 0$ is needed.

$$\varepsilon\left(\left\langle\mathbf{E}.\mathbf{E}\right\rangle\right) = \varepsilon_0'(f) + \psi(f) \tag{6}$$

where

$$\varepsilon_0'(f) = \varepsilon_0 + \phi\left(\left\langle\frac{k(0)}{k(f)}\frac{E_{00}^2}{2f^2}\right\rangle\right)$$

$$\psi(f) = \phi(\left\langle\mathbf{E}.\mathbf{E}\right\rangle .) - \phi\left(\left\langle\frac{k(0)}{k(f)}\frac{E_{00}^2}{2f^2}\right\rangle\right)$$

Here f is the dimensionless beam-width parameter defined below and k is propagation constant. Using WKB approximation and following Akhmanov et al. [21] and Sodha et al. [22] one can write

$$E(r,z) = A(r,z)\left[\frac{k(0)}{k(z)}\right]^{1/2} \exp\left[-i\int k(f)\, dz\right] \tag{7}$$

Further, we express

$$k(f) = \frac{\omega}{c}[\varepsilon_0'(f)]^{1/2} \tag{8}$$

and

$$k(0) = \frac{\omega}{c}\left[\varepsilon_0'(f=1)\right]^{1/2} \tag{9}$$