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Remote entanglement for quantum networks

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ABSTRACT

Quantum networks are distributed many-body quantum systems with tailored topology and controlled information exchange. We present two schemes to generate remote entanglement, in atomic external degrees of freedom and between cavities. In the first scheme, we entangle two atoms with their cavities in momentum space through Bragg diffraction. Thereafter, in order to trace out the cavities, we let resonantly interact an auxiliary atom with each cavity. In the last, we perform quantum measurement on two auxiliary atoms and get remote entangled state in atomic external degrees of freedom. In the second scheme, we have a three cavities system. The other two cavities, *A* and *B*, are entangled with indistinguishable modes of cavity, *C*. Performing quantum measurement on third cavity, *C*, we disentangle it from the system and the cavities, *A* and *B*, become entangled.

1. Introduction

Entanglement [1,2], a non-local and quantum mechanical phenomenon, has many applications in quantum information and quantum computation [3,4]. Different schemes are proposed to engineer different types of entanglement such as Bell state, Noon state, Werner state, Cluster state, and graph state [5–8]. Entanglement engineering between two electromagnetic cavities, multimodes of single cavity [9,10] and atomic internal [11–14] and external degrees of freedom using Bragg diffraction regime [15] are also suggested. Bragg diffraction of the atomic de Broglie waves from cavity field [16,17] has many applications in quantum informatics including quantum logic gates [18], atoms interferometer [19] and quantum state measurement [20].

The remote entanglement for field and atomic state preparation are also proposed [21]. Here, we suggest two techniques to develop remote entanglement based on cavity quantum electrodynamics (QED). In the first technique, we engineer remote entanglement in the atomic external degrees of freedom using Bragg diffraction cavity QED techniques [22,23]. For this purpose, we utilize two sets of atoms, two atoms, T_1 and T_2, used to produce momentum entangled state with cavities, called tagged atoms and other two, A_1 and A_2, used to trace out cavities from system, called auxiliary atoms. First, we interact two tagged atoms dispersively through cavities, *A* and *B*, which are in superposition state of zero and one photon. We follow dispersively interact the tagged atoms to avoid the spontaneous emission which causes the emission of photon in any arbitrary direction. After interaction we entangle the atom in their momentum states and cavity field [24]. Second, we resonantly interact the auxiliary atoms, A_1 and A_2, in the ground states, $|g_1\rangle$ and $|g_2\rangle$, with cavities, *A* and *B*, respectively. For interaction time corresponding to, π, Rabi cycle, cavity state transfer to the atomic state and leave the cavity in the vacuum state. During the interaction, when cavity is in vacuum state then auxiliary atom remain in their ground state and for one photon state the atom get the photon and goes to the excited state. Hence, we transfer atom–field entanglement to atom–atom entanglement. In the last, we perform measurement process on auxiliary atoms and get remote momentum entangled state between tagged atoms. In the second technique, we engineer remote entanglement between cavity field states. The scheme consists of three partite systems Alice (*A*), Bob (*B*) and Charles (*C*). The two cavities, *A* and *B*, are entangled with two modes of third cavity *C*, named as, C_1 and C_2 [9]. Here, the two modes of cavity, *C*, are indistinguishable. We perform quantum measurement at third party Charles, disentangle Charles from our system and engineer entanglement between two remote parties, *A* and *B*. Hence, we develop a channel between, *A* and *B*, which enable to share any information.

The paper is organized as follow: In Section 2, we entangled tagged atom in their external degrees of freedom with their respective cavities. Thereafter, we resonantly interact an auxiliary atom with each cavity to transfer cavity state to atomic state. Later, we perform measurement process on auxiliary atoms. In Section 3, we

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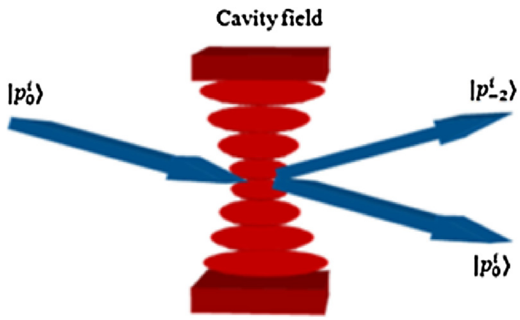

Fig. 1. The dispersive interaction of atom with cavity field in the superposition state of zero and one photon. For zero photon state their momentum remains same and for one photon state they gets a momentum kick.

engineer entanglement between cavity, A and mode C_1, of cavity C and cavity, B and mode C_2, of cavity, C. Here, cavity, C, has two indistinguishable modes, C_1 and C_2 . In the last, we perform measurement on cavity, C, and get remote entangled state between, A and B. In Section 4, we provide a general discussion of our results and experimental parameters for the proposed scheme.

2. Remote entanglement in atomic external degrees of freedom

The technique we present for engineering of remote entanglement in atomic degrees of freedom has two cavities prepared initially in superposition state, i.e. $(|0\rangle+|1\rangle/\sqrt{2})$ [15]. We have two levels tagged atoms, T_1 and T_2, both initially in their ground state with momentum states, $|P_0^i\rangle$, and two auxiliary atoms in their ground states, $|g_1\rangle$ and $|g_2\rangle$. Here, i=1, 2 stand for two atoms, T_1 and T_2. First, we interact tagged atoms with cavity fields in dispersive fashion. The initial state vector of the system before interaction is

$$|\Psi(0)\rangle = \frac{1}{2}\sum_{i=1,2}(|0_i\rangle+|1_i\rangle)\otimes|g_i,\ P_0^{(i)}\rangle.$$

The total Hamiltonian governing this interaction of an atom of mass, M, and center of mass momentum, P, with the field in the dipole and rotating wave approximation [4] is

$$\hat{H} = \frac{P^2}{2M}+\frac{\hbar\delta\,\hat{\sigma}_Z}{2}+\hbar\mu\cos(k\hat{x})[\hat{\sigma}_{eg}\hat{c}+\hat{c}^{\dagger}\hat{\sigma}_{ge}].$$

Here, $\hat{\sigma}_{ge}=|\mathrm{e}\rangle\langle\mathrm{g}|(\hat{\sigma}_{eg}=|\mathrm{g}\rangle\langle e|)$ is atomic raising (lowering) operator, $\hat{\sigma}_Z=(|e\rangle\langle e|-|g\rangle\langle g|)$ is inversion operator, $\hat{c}$ $(\hat{c}^{\dagger})$ is field annihilation (creation) operator, δ is atom–field detuning and, μ is vacuum Rabi frequency (Figs. 1 and 2).

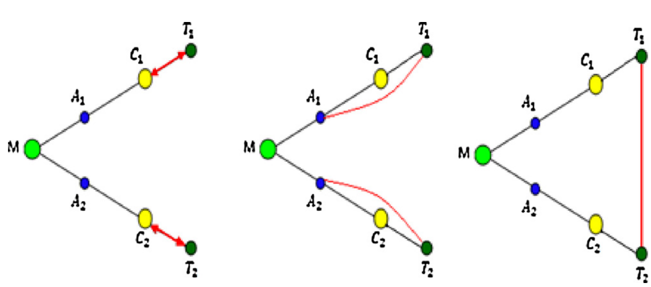

Fig. 2. The interaction of auxiliary atoms with respective cavities and the final entangled state after measurement. Here, the circles T_1 and T_2 show the tagged atoms, circles C_1 and C_2 show cavities, small Circles A_1 and A_2 show auxiliary atoms and circle, M, shows measurement operator. (a) Entanglement of tagged atoms and cavities, (b) interaction of auxiliary atoms with cavities which entangled auxiliary atoms with tagged atoms, and (c) the final entangled state between tagged atoms after performing measurement operator.

For any arbitrary time, t, the Bragg atom–field interaction wave function is [26]

$$|\Psi(t)\rangle = e^{-i\left(P_0^2/2M-\delta/2\right)}\sum_{\xi=-\infty}^{\infty}\left[A_{0,g}^{P_\xi}(t)|o,\ g,\ P_\xi\rangle+A_{1,g}^{P_\xi}(t)|1,\ g,\ P_\xi\rangle\right.$$
$$\left.+A_{0,e}^{P_\xi}(t)|0,\ e,\ P_\xi\rangle\right]. \quad (1)$$

We follow dispersive interaction to decrease the probability of de-coherence and give us only two discrete atomic momentum path. When atom with momentum, $|P_0^i\rangle$, interact with cavity in zero photon state, their momentum remains the same, i.e., $|P_0^i\rangle$, and inverted momentum state, $|P_{-2}^i\rangle$, when cavity is in one photon state [15]. The global phase factor has been introduced for mathematical convenience. Under the condition of adiabatic approximation [25] the state vector given in Eq. (1) for an interaction time, $2\pi\delta/\mu^2$ [24], gives the following entangle state.

$$|\Psi(t)\rangle = \frac{1}{2}\sum_{i=1,2}\left[|0_i,\ P_0^{(i)}\rangle+|1_i,\ P_{-2}^{(i)}\rangle\right].$$

Now to trace out cavities from the system, we interact auxiliary atoms, A_1 and A_2, initially in their ground states, $|g_1\rangle$ and $|g_2\rangle$, with cavity field. For this purpose, we resonantly interacts Auxiliary atom, A_1, with first cavity and auxiliary atom, A_2, with second cavity. The auxiliary atoms interact with optical cavities transform the atom–field entanglement to atom-atom entanglement [26]. The initial state vector of the system before interaction is

$$|\Psi(t_1)\rangle = \left[\frac{1}{\sqrt{2}}(|0_1,P_0^{(1)}\rangle+|1_1,P_{-2}^{(1)}\rangle)\otimes|\ g_1\rangle\right]$$
$$\otimes\left[\frac{1}{\sqrt{2}}(|0_2,P_0^{(2)}\rangle+|1_2,P_{-2}^{(2)}\rangle)\otimes|\ g_2\rangle\right].$$

First, we resonantly interacts first auxiliary atom, A_1, with first cavity under the Hamiltonian, $\hat{H}=\hbar\mu(\hat{\sigma_{g_1e_1}}\hat{c}+\hat{\sigma_{e_1g_1}}\hat{c}^{\dagger})$. Here, μ is atom–field coupling constant, $\hat{\sigma_{g_1e_1}}(\hat{\sigma_{e_1g_1}})$ denotes the atomic raising (lowering) operators and, $\hat{c}$ and $\hat{c}^{\dagger}$ are field annihilation and creation operators. After the interaction of the first auxiliary the wave function for the system is

$$|\Psi(t_1)\rangle = \frac{1}{2}[\{|0_1,\ g_1,\ P_0^{(1)}\rangle+\cos(\mu_r t_1)|1_1,\ g_1,\ P_{-2}^{(1)}\rangle$$
$$-i\sin(\mu_r t_1)|0_1,\ e_1,\ P_{-2}^{(1)}\rangle\}\otimes\{(|0_2,\ P_0^{(2)}\rangle+|1_2,\ P_{-2}^{(2)}\rangle)$$
$$\otimes|g_2\rangle\}].$$

For an interaction time, $t_1=\pi/2\mu_r$ [24], the state of first cavity transfer to the first auxiliary atom, A_1, and first cavity comes to vacuum state. The state vector for the system is

$$|\Psi(t_1)\rangle = \frac{1}{2}[(|g_1,\ P_0^{(1)}\rangle-i|e_1,\ P_{-2}^{(1)}\rangle)|0\rangle\otimes(|0_2,\ P_0^{(2)}\rangle+|1_2,\ P_{-2}^{(2)}\rangle)|g_2\rangle].$$

Now, we interact second auxiliary atom, A_2, resonantly with the second cavity field. For a time of interaction equal to, π, Rabi cycle [26], i.e. $t_2=\pi/(2\mu_r)$ the second cavity state transfer to second auxiliary atom and comes to vacuum state. The final state of the system becomes

$$|\Psi(t_2)\rangle = \frac{1}{2}[(|g_1,\ P_0^{(1)}\rangle-i|e_1,\ P_{-2}^{(1)}\rangle)\otimes(|g_2,\ P_0^{(2)}\rangle-i|e_2,\ P_{-2}^{(2)}\rangle]|0\,0\rangle.$$

The combined state of the system can be written as