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First-order kernel density estimation of abdomen medical image intensity and spatial information and application to segmentation

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ABSTRACT

Kernel density estimators (KDE) used for many medical image applications only consider the intensity information of each pixel or its neighbors without the ability of expressing the structure and shape of tissues and organs, and they suffer from boundary bias problem. In this paper, we propose a new first-order kernel density estimation (FOKDE) method for 1D intensity information and 2D spatial information of medical image in two steps. First, the FOKDE of intensity information is estimated and applied to medical image segmentation with the multi-thresholding algorithm. Second, we estimate the FOKDE of spatial information on the initial segmentation, which can express the structure and shape of organs and tissues. In order to evaluate the FOKDE and KDE of the 2D spatial information, we apply them to medical image segmentation with the hill-climbing strategy. Density estimation experiments and segmentation application results on the simulated dataset and real abdomen CT images show us that the FOKDE has smaller boundary bias than the KDE, and that it can estimate the structure and shape of tissues and organs with spatial information effectively.

1. Introduction

It is important to model the unknown density functions of the images in medial image processing system [1]. The density function can be applied to medical image segmentation [2,3] in which the density estimate can be used as a priori probability. Medical image registrations [4,5] are achieved frequently through many quantities derived from the density function, particularly entropy and mutual information. Many medical image features are extracted based on the density estimate and are used for image retrieval [6], image classification [7] and image categorization [8]. Most published methods estimate the density function using histograms [9,10] or kernel density estimators [11,12] (often called Parzen Windows estimators). These approaches have the advantage of being nonparametric, so they are generally applicable.

The abdomen region images are difficult to segment because the intensities of the images are near each other and the contrast between various structures is poor. Therefore, density estimation of the spatial information is very important to medical image process applications. The most successful image segmentation approaches have been based on the consideration of the spatial information as implicit prior information [13]. The histogram and KDE in these applications [9–12] are only based on image intensity information without taking into account the spatial correlation of the same or similar valued elements. Several approaches incorporate the spatial information into the histogram and KDE. Kupinski [14] used an isotropic Gaussian function to interpret the prior knowledge of each target region's shape. They multiplied this spatial constraint function with the original image to suppress distant pixel values. Liew [15] introduced a spatial constraint where if all 9 pixel in a 3×3 patch belong to the same class, the center pixel will be smoothed by its neighbors. Cheng [16] used the fuzzy homogeneity approach in which the concept of homogram was introduced. Mohabey [17] introduced a concept of histon, which is a contour plotted on the top of the histogram by considering a similar color sphere of a predefined radius around a pixel. However, the improved histogram or KDE methods with so called spatial information, which is just the intensity information of its neighbor, are still a 1D density distribution which represents the global information [18], and they can't depict the shape and structure of tissues and organs.

The most straightforward method is to include the coordinate information as a part of features, like the idea of Greenspan's

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method [19]. However, since the coordinate data does not form a compact mass in the feature space, this method will result in loose clusters even for a perfect feature set [20] and the curse of dimensionality. First, the computational time and space burden can increase exponentially with dimension. Second, the number of samples needed grows exponentially with the increase of dimension. One slice of the medical image sequence only has the number of samples for 2D data set, which is not enough for the 3D feature space. Therefore, density estimate of the 2D medical image with three features cannot be smooth.

One motivation and contribution of this paper is to estimate the 2D density function of the medical image which can express the shape and structure of tissues and organs with the intensity information and spatial information in two steps. First, we estimate the density function of global intensity information and use it to segment initially the medical images, and then estimate the 2D density function of spatial information for each initial segmentation result. Our method can overcome the problems of density estimate with only 1D intensity information or 3D features.

Another motivation and contribution of this paper is extending the KDE to the FOKDE for 1D intensity information and 2D spatial information to reduce the boundary bias effect. Because of the noise, radical variation and the partial volume effect, the boundary biases of the KDE for tissues and organs in abdomen images are more serious than other images. To validate the FOKDE, we apply it to medical image segmentation with the multi-thresholding algorithm and hill-climbing strategy.

The paper is organized as follows. Relative works are reviewed in Section 2. Our works is presented in Section 3. Application of the FOKDE to medical image segmentation is presented in Section 4. Experiment results are shown in Section 5. Some conclusions are presented in Section 6.

2. Relevant works

Since the KDE was proposed by Parzen [21], its relative topics have been investigated over the last few decades from different perspectives. In this paper, we focus on spatial information, bias reduction and kernel bandwidth [22] in medical image process application. The relative works on first problem is reviewed in Section 1.

The bias reduction in kernel estimation has received considerable attention in the statistics literatures, such as the boundary kernel method [23]. However, a drawback of the boundary kernel method is that the estimates might be negative near the endpoints. To correct this deficiency, some remedies have been proposed. Gasser [24] has suggested various boundary kernels mixture to remove the bias. Rice [25] suggested a direct method for eliminating the second order bias using a linear combination of two estimators. Jones [26] proposed that a linear combination of a kernel and its derivative can also remove the second order bias. The local linear method is a special case of the boundary kernel method that is thought of by some as a simple, hard-to-beat default approach. The boundary kernel and related methods usually have low bias but the price for that is an increase in variance [27].

Another hot topic is the estimation of the kernel bandwidth [28]. The bandwidth influences the degree of smoothing for the density function approximation and the location of its modes. The algorithms for finding the bandwidth in statistics can be classified into two categories: quality-of-fit methods and plug-in methods. The quality-of-fit methods use cross-validation by leaving certain data samples out while approximating the density function with the sum of kernels located at the remaining data [29]. The plug-in methods calculate the bias in the density function approximation such that it minimizes the mean integrated square error (MISE) between the real density and its kernel-based approximation [30]. However, the plug-in methods require an initial pilot estimate of the bandwidth for an iterative estimation process. Botev [31] presented a new plug-in bandwidth selection method that is free from the arbitrary normal reference rules used by existing methods. Besides those methods, Adrian [32] proposed a Bayesian approach to find the kernel bandwidth by modeling distributions of variances of localized data subsets.

3. First-order kernel density estimation for medical image

3.1. FOKDE of the intensity information

Suppose the pixel intensities of medical image $X_1, X_2, \ldots, X_n$ have an unknown density $f(x)$. An extension of local likelihood method to the density estimation problem is described by Loader [33]. Consider the log-likelihood function

$$L(f)=\sum_{i=1}^{n}\log(f(X_i))-n\left(\int_{\aleph}f(u)\,\mathrm{d}u-1\right) \tag{1}$$

where $\aleph$ is the domain of the density. $n\left(\int_{\aleph}f(u)\mathrm{d}u-1\right)$ is the added a penalty term. If f is a density, the penalty is 0. The reason for adding the penalty is that $L(f)$ can be treated as a likelihood function for any non-negative function f without imposing the constraint $\int f(x)\mathrm{d}x=1$. A localized version of the log-likelihood [33] is

$$L_x(f)=\sum_{i=1}^{n}W\left(\frac{X_i-x}{h}\right)\log(f(X_i))-n\int_{\aleph}W\left(\frac{u-x}{h}\right)f(u)\,\mathrm{d}u \tag{2}$$

where $W(\bullet)$ is a kernel function and h is a smooth parameter. A local polynomial approximation for $\log[f(u)]$ in the neighborhood of x can be given by

$$g(u)=\log[f(u)]\approx a_0+a_1(u-x)+\cdots+a_p\frac{(u-x)^p}{p!} \tag{3}$$

The local likelihood is given by

$$\begin{aligned}L_x(a)=&\sum_{i=1}^{n}W\left(\frac{X_i-x}{h}\right)\left[a_0+a_1(X_i-x)+\cdots+a_p\frac{(X_i-x)^p}{p!}\right]\\&-n\int_{\aleph}W\left(\frac{u-x}{h}\right)\exp\left[a_0+a_1(u-x)+\cdots+a_p\frac{(u-x)^p}{p!}\right]\mathrm{d}u\end{aligned} \tag{4}$$

Let $\hat{a}=(\hat{a}_0,\hat{a}_1,\ldots,\hat{a}_p)^T$ be the coefficients to maximize of the local log-likelihood.

Theorem 1. For $p=0$, the local likelihood density estimate in Eq. (4) is the KDE.

Proof: according to Eq. (4), for $p=0$, the local likelihood density estimate is

$$L_x(a)=\sum_{i=1}^{n}W\left(\frac{X_i-x}{h}\right)a_0-n\int_{\aleph}W\left(\frac{u-x}{h}\right)\exp(a_0)\mathrm{d}u \tag{5}$$

The local parameter vector $\hat{a}$ is the solution of the system of local likelihood equations obtained by differentiating Eq. (5). Because $\hat{a}=(\hat{a}_0,\hat{a}_1,\ldots,\hat{a}_p)^T$ are the coefficients that maximize of the local log-likelihood, we have

$$\frac{\partial L_x(a)}{\partial a_0}=\sum_{i=1}^{n}W\left(\frac{X_i-x}{h}\right)-n\exp(a_0)\int W\left(\frac{u-x}{h}\right)\mathrm{d}u=0 \tag{6}$$