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Generation and circuit implementation of multi-block multidirectional grid multi-scroll chaotic attractors

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ABSTRACT

Due to the dynamic characteristics of the Chua's system, multi-scroll chaotic attractors are still confined in a single block and fail to break the limit. This paper proposes an approach for generating novel multi-block multidirectional grid multi-scroll chaotic attractors that can break the limit via novel nonlinear modulating functions. According to this method, the recursion rules used to generate multi-block multi-directional grid multi-scroll attractors are mathematically obtained. The new system is autonomous; the effectiveness of this method has been verified by theoretical analysis, numerical simulation, and circuit implementation.

1. Introduction

In nonlinear system and circuit area, producing all kinds of different types of chaotic and hyperchaotic signals which are suitable for secure communications, digital encryption is becoming a hot topic in physics and information science in recent years [1–3]. In 1993, Suykens et al. [4] first constructed piecewise nonlinear function on the basis of Chua's system for generating multi-scroll attractors in a single direction. Then people have proposed different nonlinear functions to generate multi-scroll [5–9], studies for generating multi-scroll chaotic attractor have been becoming mature which drives the chaos system to generate a relevant number of scrolls through changing the number of turning points in the nonlinear functions. For example, Yalcin et al. [10] proposed an approach for using step function for generating 1-D n-scroll, 2-D $n \times m$-grid scroll, and 3-D $n \times m \times l$-grid scroll chaotic attractors; In 2007, Yu et al. [11] proposed the general Chua's grid multi-scroll chaotic circuit, using the circuit for generating n-scroll and $n \times m$-grid multi-scroll chaotic attractors; Lü et al. [12] analyzed the reason for generating multi-scroll in detail by using the nonlinear theory where it can be seen that the main design criterion of conventional approach for generating grid multi-scroll chaotic system lies in the choices of an appropriate double-scroll chaotic system and nonlinear function, from which the number of index 2 saddle-focus equilibrium points can be increased and extended along a certain plane or space. In 2013, Xu et al. [13] proposed a novel approach for constructing a high-order Chua's circuit which can generate multidirectional multi-scroll chaotic attractors can be realized by introducing a RC structure and suitable step functions. Based on the Chua's system, by constructing odd-symmetric multi-segment linear functions [14], saw tooth wave [15,16], triangle wave [17], step wave [18], hysteresis sequence [19], and saturated sequence [20,21], multi-scroll chaotic attractors can be obtained. However, in the existing literatures, all of the grid multi-scroll chaotic attractors are single-block, whether can construct multi-block multidirectional grid multi-scroll chaotic attractors which have more dynamical characteristics complexity and more potential engineering applications of the new design than single grid multi-scroll chaotic attractors. One may ask whether or not there exists another method to break such a single-block limitation? This Letter gives a positive answer to the question.

Motivated by the above discussion, to be specific, by changing the positions of novel nonlinear functions and adding some other novel nonlinear functions in the new system we proposed an approach which can generate multi-scroll chaotic attractors, and control the relationships of parameters between these different nonlinear functions; multi-block multidirectional grid chaotic attractors are generated, which provides some principles and guidelines for design approach on multi-block chaotic attractors. Some basic dynamical properties, such as equilibrium points, the maximum Lyapunov exponents, bifurcation diagram, and chaotic dynamical behaviors of the new chaotic system are investigated. Based on theoretical analysis and numerical simulations, chaos circuit design and hardware experiment are carried out, the

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Table 1
Corresponding parameters of two-attractor $4 \times 3 \times 3$ scroll.

$f_i(u, \xi_1)$	ξ_1	M_i	N_i
$f_1(x, \xi_1)$	$\xi_1 = 1$	$M_1 = 1$	×
$f_2(y, \xi_2)$	$\xi_2 = 1$	×	$N_2 = 1$
$f_3(z, \xi_3)$	$\xi_3 = 1$	×	$N_3 = 1$
$f_4(w, \xi_4)$	$\xi_4 = 0.4$	$M_4 = 1$	×

experimental results prove the consistency of numerical simulation and circuit experiment results.

The rest of this paper is organized as follows: In Section 2, new step functions are introduced and some fundamental conditions of diagonal directional chaos generation are discussed for a given four-dimensional linear autonomous system, then a new systematic method for generating multi-block multidirectional grid multi-scroll chaotic attractors, including 4-block multidirectional grid $4 \times 3 \times 3$-multi-scroll chaotic attractors, 3-block multidirectional grid $3 \times 5 \times 6$-multi-scroll chaotic attractors are presented based on a given three-dimensional linear autonomous system. The nonlinear dynamics of the chaotic attractors are analyzed using bifurcation diagrams and the maximum Lyapunov exponent spectrum in Section 3. A module-based circuit design and circuit implementation observations are demonstrated in Section 4. Conclusions are finally drawn in Section 5.

2. Generating multi-block multidirectional grid multi-scroll chaotic attractors

2.1. Diagonal directional multi-scroll chaotic attractors

In order to generate four-dimensional butterfly chaotic attractors, it is found to be essential to design additional index-2 equilibrium points based on two existing index-2 equilibrium points. By introducing two differential step functions, a novel diagonal directional multi-scroll chaotic system is described as

$$\begin{cases} \dfrac{dx}{dt} = aw + (y - f_2(y, \xi 2)), \\ \dfrac{dy}{dt} = b(w - f_4(w, \xi 4)), \\ \dfrac{dz}{dt} = cw - (z - f_3(z, \xi 3)), \\ \dfrac{dw}{dt} = dw - (x - f_1(x, \xi 1)), \end{cases} \tag{1}$$

where x, y, z, w are state variables, $a = 1$, $b = 2$, $c = 1.5$, $d = -1$, $f_1(x, \xi_1), f_2(y, \xi_2), f_3(z, \xi_3), f_4(w, \xi_4)$ are step functions defined by

$$f_i(u, \xi_i) = \xi_i \sum_{j=-M_i}^{M_i} sgn[u + 2j\xi_i], \tag{2}$$

$$f_i(u, \xi_i) = \xi_i \sum_{\substack{j=-N_i \\ J \neq 0}}^{N_i} sgn[u + \xi_i(2j - \frac{|j|}{j})], \tag{3}$$

where i, j denote natural numbers and $u \in \{x, y, z, w\}$, $1 \le i \le 4$, which are adjustable parameters when $u = x, u = y, u = z, u = w$, respectively, which determine the maximum number of chaotic scrolls, $M_i \ge 0$, $N_i \ge 0$. Corresponding parameters of the nonlinear functions $f1(x, \xi 1), f2(y, \xi 2), f2(y, \xi 2), f3(z, \xi 3), f4(w, \xi 4)$ are listed in Table 1. The numerical simulation results are shown in Fig. 1.

The generation of $k \times n \times m$-scroll attractor is inspired by the idea of increasing the equilibrium points of the system, through adding nonlinear function into three-order Jerk system, it is easy to obtain multi-scroll chaotic attractor in a single direction, its mathematical expression is modified as follows:

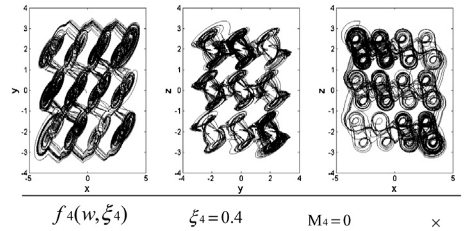

Fig. 1. Two-attractor $4 \times 3 \times 3$-scroll chaotic attractors.

$$\begin{cases} \dot{x} = y, \\ \dot{y} = z, \\ \dot{z} = \alpha\left(-x - y - z + f_1(x, \xi_1)\right), \end{cases} \tag{4}$$

where $\alpha = 0.7$, $f_1(x, \xi_1)$ is a step function. Now, we delete $f_1(x, \xi_1)$ and add $f_2(y, \xi_2)$ into system (4), the system (4) is now modified as follows:

$$\begin{cases} \dot{x} = y - f_2(y, \xi_2), \\ \dot{y} = z, \\ \dot{z} = \alpha(-x - y - z), \end{cases} \tag{5}$$

where $\alpha = 0.7$, $f_2(y, \xi_2)$ is given by the nonlinear function (3) where corresponding parameters: $\xi_2 = 0.5$, $N_2 = 1$, numerical simulation result of 3-scroll chaotic attractor in oblique direction is as shown in Fig. 2(a). Besides, when $\alpha = 0.7$, $f_2(y, \xi_2)$ is given by the nonlinear function (2) and corresponding parameters: $\xi_2 = 0.5$, $M_2 = 1$, a 4-scroll chaotic attractor can be obtained as shown in Fig. 2(b).

2.2. Generation of multi-block multidirectional grid multi-scroll chaotic attractors

In order to generate grid multi-scroll chaotic attractors, it is found to be essential to design a grid of index-2 equilibrium points. By adding a step function $f_3(z, \xi_3)$ into system (5), it is easy to

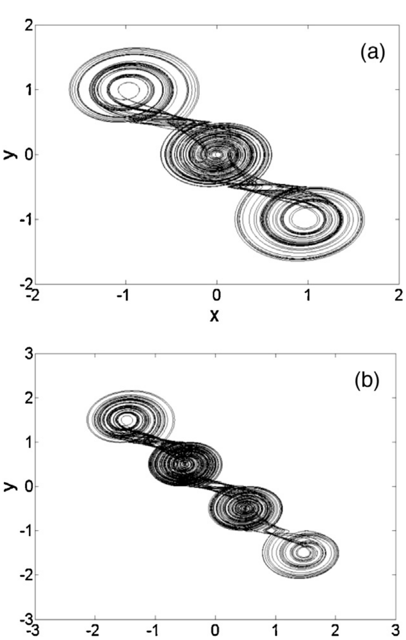

Fig. 2. (a) 3-scroll chaotic attractor, (b) 4-scroll chaotic attractor.