Optik 127 (2016) 127–130

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

A new quantum approach of one-dimensional photonic crystals

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ARTICLE INFO

Article history:
Received 17 December 2014
Accepted 10 October 2015

PACS:
42.70.Qs
78.20.Ci
41.20.Jb

Keywords:
Photonic crystals
Quantum matrix
Quantum transmissivity

ABSTRACT

In this paper, we have presented a quantum theory to study one-dimensional photonic crystals, and give the quantum transform matrix and quantum transmissivity. We calculate the quantum transmissivity with defect layer, which include absorbing medium and active medium, and obtain some valuable results. The quantum approach can be used to study two-dimensional and three-dimensional photonic crystals.

1. Introduction

In 1987, E. Yablonovitch and S. John had pointed out that the behavior of photons can be changed when propagating in the material with periodical dielectric constant, and termed such material Photonic Crystal [1,2]. Photonic crystal important characteristics are: Photon Band Gap, defect states, Light Localization and so on. These characteristics make it able to control photons, so it may be used to manufacture some high performance devices which have completely new principles or can not be manufactured before, such as high-efficiency semiconductor lasers, right emitting diodes, wave guides, optical filters, high-Q resonators, antennas, frequency-selective surface, optical wave guides and sharp bends [3,4], WDM-devices [5,6], splitters and combiners [7,8]. optical limiters and amplifiers [9,10]. The research on photonic crystals will promote its application and development on integrated photoelectron devices and optical communication. To investigate the structure and characteristics of band gap, there are many methods to analyze Photonic crystals including the plane-wave expansion method [11], Greens function method, finite-difference time-domain method [12–14] and transfer matrix method [15–17]. All of methods are come from classical Maxwell equations. In this paper, We should study the 1D Photonic crystals by the quantum wave equations of photon [18,19], and give quantum transmissivity. In numerical calculation, we calculate the quantum transmissivity with and without defect layer, which include absorbing medium and active medium, and obtain some valuable results. The quantum approach can be used to study two-dimensional and three-dimensional photonic crystals.

2. The quantum wave equation and probability current density of photon

The quantum wave equations of free and non-free photon have been obtained in Refs. [18,19], they are

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t)=c\hbar\nabla\times\vec{\psi}(\vec{r},t), \quad (1)$$

and

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t)=c\hbar\nabla\times\vec{\psi}(\vec{r},t)+V\vec{\psi}(\vec{r},t), \quad (2)$$

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http://dx.doi.org/10.1016/j.ijleo.2015.10.026

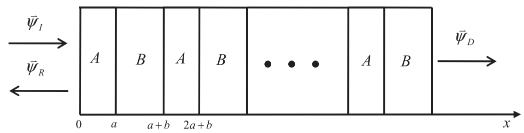

Fig. 1. The structure of one-dimensional photonic crystals.

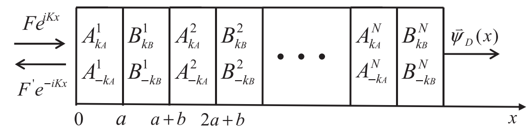

Fig. 2. The quantum structure of one-dimensional photonic crystals.

where $\vec{\psi}(\vec{r}, t)$ is the vector wave function of photon, and V is the potential energy of photon in medium. In the medium of refractive index n, the photon's potential energy V is [18,19]

$$V = \hbar\omega(1 - n). \tag{3}$$

By Eq. (2), we obtain

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0, \tag{4}$$

where

$$\rho = \vec{\psi}^* \cdot \vec{\psi}, \tag{5}$$

and

$$J = ic\vec{\psi} \times \vec{\psi}^*, \tag{6}$$

are the probability density and probability current density, respectively. By the method of separation variable

$$\vec{\psi}(\vec{r}, t) = \vec{\psi}(\vec{r}) f(t), \tag{7}$$

the time-dependent Eq. (2) becomes the time-independent equation

$$c\hbar\nabla \times \vec{\psi}(\vec{r}) + V\vec{\psi}(\vec{r}) = E\vec{\psi}(\vec{r}), \tag{8}$$

where E and V are the energy and potential energy of photon in medium, respectively.

3. The quantum transmissivity

We consider the photon travels along with the x axis in one-dimensional Photonic crystals, which is shown in Fig. 1. The wave vector $k_y = k_z = 0$ and $k_x \neq 0$. Since the photon wave is transverse wave, we have

$$\begin{cases} \psi_x = 0 \\ \psi_y = \psi_{0y} e^{i(kx-\omega t)} \\ \psi_z = \psi_{0z} e^{i(kx-\omega t)} \end{cases}, \tag{9}$$

where $k = \frac{\omega}{c} n$ is the wave vector of photon in medium. Substituting Eq. (9) into Eq. (8), we get

$$\begin{cases} -ik\psi_z = \dfrac{\omega}{c} n\psi_y = k\psi_y \\ ik\psi_y = \dfrac{\omega}{c} n\psi_z = k\psi_z \end{cases}, \tag{10}$$

their solution are

$$\psi_y = \psi_{0y} e^{i\frac{\omega}{c}nx} = \psi_{0y} e^{ikx} \tag{11}$$

and

$$\psi_z = \psi_{0z} e^{ikx} = i\psi_y = i\psi_{0y} e^{ikx} \tag{12}$$

the total wave function of photon in medium is

$$\vec{\psi} = \psi_y \vec{j} + \psi_z \vec{k} = \psi_{0y} e^{ikx} \vec{j} + \psi_{0z} e^{ikx} \vec{k}. \tag{13}$$

the total wave function of photon in vacuum is

$$\vec{\psi} = \psi_y \vec{j} + \psi_z \vec{k} = \psi_{0y} e^{iKx} \vec{j} + \psi_{0z} e^{iKx} \vec{k}. \tag{14}$$

where $K = \frac{\omega}{c}$ is the wave vector of photon in vacuum.

For one-dimensional Photonic crystals, we should define and calculate its quantum transmissivity and quantum reflectivity. The one-dimensional photonic crystals structure is shown in Fig. 1.

In Fig. 1, $\vec{\psi}_I$, $\vec{\psi}_R$, $\vec{\psi}_D$ are the wave functions of incident, reflection and transmission photon, respectively, they can be written as

$$\vec{\psi}_I = F_y e^{i(\vec{k}\cdot\vec{r}-\omega t)} \vec{j} + F_z e^{i(\vec{k}\cdot\vec{r}-\omega t)} \vec{k}, \tag{15}$$

$$\vec{\psi}_R = F'_y e^{i(\vec{k}\cdot\vec{r}-\omega t)} \vec{j} + F'_z e^{i(\vec{k}\cdot\vec{r}-\omega t)} \vec{k}, \tag{16}$$

$$\vec{\psi}_D = D_y e^{i(\vec{k}\cdot\vec{r}-\omega t)} \vec{j} + D_z e^{i(\vec{k}\cdot\vec{r}-\omega t)} \vec{k}, \tag{17}$$

where F_y, F_z, F'_y, F'_z, D_y, and D_z are incident, reflection and transmission amplitudes of y and z components.

By Eq. (13), the probability current density can be written as

$$J = ic\vec{\psi} \times \vec{\psi}^* = 2c|\psi_z|^2 \vec{i} = 2c|\psi_{0z}|^2 \vec{i}, \tag{18}$$

where ψ_{0z} is the amplitude of $\psi_z = \psi_{0z} e^{i(\vec{k}\cdot\vec{r}-\omega t)}$.

For the incident, reflection and transmission photon, their probability current density J_I, J_R, J_T are

$$J_I = 2c|F_z|^2, \qquad J_R = 2c|F'_z|^2, \qquad J_T = 2c|D_z|^2, \tag{19}$$

We can define quantum transmissivity T is

$$T = \frac{J_T}{J_I} = \left|\frac{D_z}{F_z}\right|^2, \tag{20}$$

By the amplitudes of z component F_z and D_z, we can calculate the quantum transmissivity.

4. The photon wave function and quantum transform matrix in one-dimensional photonic crystals

In Fig. 2, we give the simplification form of wave function in every medium, such as symbols $A^1_{k_A}$ and $A^1_{-k_A}$ express simplifying wave function of medium A in the first period, it express wave function

$$\psi^1_A(x) = A^1_{k_A} e^{ik_A x} + A^1_{-k_A} e^{ik_A a + ik_A(a-x)}, \tag{21}$$

in medium B of first period, the symbols $B^1_{k_B}$ and $B^1_{-k_B}$ express wave function

$$\psi^1_B(x) = B^1_{k_B} e^{ik_A a + ik_B(x-a)} + B^1_{-k_B} e^{ik_A a + ik_B b + ik_B(a+b-x)}, \tag{22}$$

similarly, in medium A of Nth period, the symbols $A^N_{k_A}$ and $A^N_{-k_A}$ express wave function

$$\psi^N_A(x) = A^N_{k_A} e^{ik_A(N-1)a + ik_B(N-1)b + ik_A(x-(N-1)a-(N-1)b)} + A^N_{-k_A} e^{ik_A Na + ik_B(N-1)b + ik_A(Na+(N-1)b-x)}, \tag{23}$$

in medium B of Nth period, the symbols $B^N_{k_B}$ and $B^N_{-k_B}$ express wave function

$$\psi^N_B(x) = B^N_{k_B} e^{ik_A Na + ik_B(N-1)b + ik_B(x-Na-(N-1)b)} + B^N_{-k_B} e^{ik_A Na + ik_B Nb + ik_B(Na+Nb-x)}. \tag{24}$$

In the incident area, the total wave function $\psi_{tot}(x)$ is the superposition of incident and reflection wave function, it is

$$\psi_{tot}(x) = \psi_I(x) + \psi_R(x) = Fe^{iKx} + F'e^{-iKx}, \tag{25}$$