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Optical soliton solutions of nonlinear evolution equations using ansatz method

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ABSTRACT

In this paper, the optical soliton solutions are obtained for the modified Korteweg de Vries-type equation and the dissipative (2+1) dimensional Ablowitz–Kaup–Newell–Segur (AKNS) equation. The mathematical techniques used to obtain the soliton solution is the ansatz method.

1. Introduction

Optical solitons and solitary waves are most important branches of study in the field of nonlinear waves. In particular, optical solitons are pulses that act as information carriers through optical fibers for trans-continental and trans-oceanic distances. These solitons form the fabric of our daily lives in the Internet word and other forms of global electronic communications across trans-continental and trans-oceanic distances. The results of such research will always lead to the bleeding edge technology in this field. There have been a lot of research activities in this area. A plethora of papers has been published in this field for the past few decades [1–5].

The construction of exact traveling wave solutions of nonlinear evolution equations (NEEs) is one of the most important and essential tasks in nonlinear science, since these solutions will very well describe various natural phenomena, such as vibrations, solitons, and propagation with a finite speed. The rapid developments of nonlinear sciences, a wide range of straightforward and effective methods have been introduced to obtain traveling wave solutions of NEEs in [6–14].

The integrability aspect and exact solutions will be the focus of this paper. The aim is to extract dark soliton solution to the NEEs. While there are several integration tools that are available to solve such problems, this paper will address the ansatz approach. This is indeed a powerful as well as popular integration architecture that has gained fame in the past few years. The computer symbolic systems such as Maple and Mathematica allow us to perform merciless and unforgiving calculations.

2. The modified Korteweg de Vries-type equation

The mKdV-type equation is given by [15]

$$uu_{xxt} - u_x u_{xt} - 4u^3 u_t + 4uu_{xxx} - 4u_x u_{xx} - 16u^3 u_x = 0, \tag{1.1}$$

where u is a real-valued scalar function, t is time and x is a spatial variable. In [16], Wazwaz found a variety of travelling wave solutions such as kinks, solitons, peakons, periodic, etc. this equation.

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Bogning studied by the Bogning–Djeumen Tchaho–Kofané method to look for all solutions of Eq. (1.1) [17].

In order to start off with the solution hypothesis, the following ansatz is assumed [18–28],

$$u(x,t) = \lambda \tanh^p \tau, \tag{1.2}$$

and

$$\tau = \eta(x - vt) \tag{1.3}$$

where the λ and η are the free parameters, and v is the velocity of the soliton. The exponent p is also unknown. These will be determined

From Eqs. (1.2) and (1.3), we have:

$$uu_{xxt} = -pv\lambda^2\eta^3 \left\{ \begin{array}{l} (p-1)(p-2)(\tanh^{2p-3}\tau - \tanh^{2p-1}\tau) \\ +(p+1)(p+2)(\tanh^{2p+1}\tau - \tanh^{2p+3}\tau) \\ +2p^2(\tanh^{2p+1}\tau - \tanh^{2p-1}\tau) \end{array} \right\}, \tag{1.4}$$

$$u^3u_x = p\lambda^4\eta\{\tanh^{4p-1}\tau - \tanh^{4p+1}\tau\}, \tag{1.5}$$

$$u^3u_t = pv\lambda^4\eta\{\tanh^{4p+1}\tau - \tanh^{4p-1}\tau\}, \tag{1.6}$$

$$u_xu_{xx} = p^2\lambda^2\eta^3 \left\{ \begin{array}{l} (p-1)\tanh^{2p-3}\tau - (3p-1)\tanh^{2p-1}\tau \\ +(3p+1)\tanh^{2p+1}\tau - (p+1)\tanh^{2p+3}\tau \end{array} \right\}, \tag{1.7}$$

$$uu_{xxx} = p\lambda^2\eta^3 \left\{ \begin{array}{l} (p-1)(p-2)\tanh^{2p-3}\tau - [2p^2 + (p-1)(p-2)]\tanh^{2p-1}\tau \\ +[2p^2 + (p+1)(p+2)]\tanh^{2p+1}\tau - (p+1)(p+2)\tanh^{2p+3}\tau \end{array} \right\}, \tag{1.8}$$

$$u_xu_{xt} = vp^2\lambda^2\eta^3 \left\{ \begin{array}{l} -(p-1)\tanh^{2p-3}\tau + (3p-1)\tanh^{2p-1}\tau \\ -(3p+1)\tanh^{2p+1}\tau + (p+1)\tanh^{2p+3}\tau \end{array} \right\}, \tag{1.9}$$

where $\tau = \eta(x - vt)$. Substituting Eqs. (1.4)–(1.9) into Eq. (1.1), we obtain:

$$\begin{array}{rl} & -pv\lambda^2\eta^3 \left\{ \begin{array}{l} (p-1)(p-2)(\tanh^{2p-3}\tau - \tanh^{2p-1}\tau) \\ +(p+1)(p+2)(\tanh^{2p+1}\tau - \tanh^{2p+3}\tau) \\ +2p^2(\tanh^{2p+1}\tau - \tanh^{2p-1}\tau) \end{array} \right\} \\ & -vp^2\lambda^2\eta^3 \left\{ \begin{array}{l} -(p-1)\tanh^{2p-3}\tau + (3p-1)\tanh^{2p-1}\tau \\ -(3p+1)\tanh^{2p+1}\tau + (p+1)\tanh^{2p+3}\tau \end{array} \right\} \\ & -4pv\lambda^4\eta\{\tanh^{4p+1}\tau - \tanh^{4p-1}\tau\} \\ & +4p\lambda^2\eta^3 \left\{ \begin{array}{l} (p-1)(p-2)\tanh^{2p-3}\tau - [2p^2 + (p-1)(p-2)]\tanh^{2p-1}\tau \\ +[2p^2 + (p+1)(p+2)]\tanh^{2p+1}\tau - (p+1)(p+2)\tanh^{2p+3}\tau \end{array} \right\} \\ & -4p^2\lambda^2\eta^3 \left\{ \begin{array}{l} (p-1)\tanh^{2p-3}\tau - (3p-1)\tanh^{2p-1}\tau \\ +(3p+1)\tanh^{2p+1}\tau - (p+1)\tanh^{2p+3}\tau \end{array} \right\} \\ & -16p\lambda^4\eta\{\tanh^{4p-1}\tau - \tanh^{4p+1}\tau\} \\ = & 0. \end{array} \tag{1.10}$$

Now, from (1.10) equating the exponents of $\tanh^{4p+1}\tau$ and $\tanh^{2p+3}\tau$ gives,

$$4p + 1 = 2p + 3, \tag{1.11}$$

so that

$$p = 1. \tag{1.12}$$

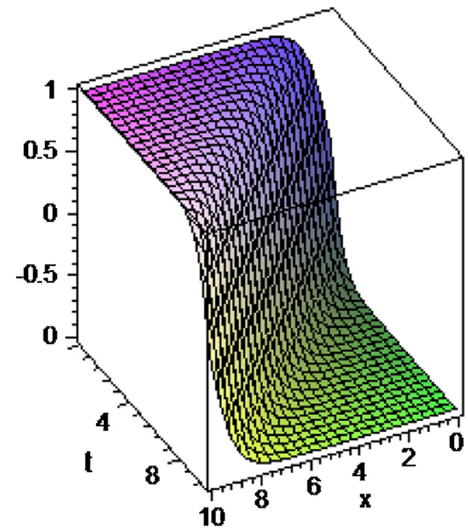

Fig. 1. The exact solution $u_{1,2}(x, t)$ for Eq. (1.1) when $p = 1$, $\lambda = 1$, $v = 1$.

It needs to be noted that the same value of p is yielded when the exponents pair $4p - 1$ and $2p + 1$ is equated with each other. Setting their respective coefficients to zero yields a set of algebraic equations:

$$\begin{array}{rl} & p(p+1)(p+2)v\lambda^2\eta^3 - vp^2(p+1)\lambda^2\eta^3 - 4pv\lambda^4\eta \\ & -4p(p+1)(p+2)\lambda^2\eta^3 + 4p^2(p+1)\lambda^2\eta^3 + 16p\lambda^4\eta \\ = & 0, \end{array} \tag{1.13}$$

$$\begin{array}{rl} & -p(p+1)(p+2)v\lambda^2\eta^3 - 2p^3v\lambda^2\eta^3 + vp^2(3p+1)\lambda^2\eta^3 \\ & +4pv\lambda^4\eta + 4p[2p^2 + (p+1)(p+2)]\lambda^2\eta^3 \\ & -4p^2(3p+1)\lambda^2\eta^3 - 16p\lambda^4\eta \\ = & 0, \end{array} \tag{1.14}$$

which gives after some calculations and with Eq. (1.13) or (1.14) with (1.12) also we get

$$\eta = \mp\lambda. \tag{1.15}$$

Hence, finally, the 1-soliton solution to (1.1) is given by

$$u_{1,2}(x,t) = \lambda \tanh(\mp\lambda(x - vt)), \tag{1.16}$$

which exist provided that $v \neq 4$ (Figs. 1 and 2).

3. The dissipative (2+1) dimensional AKNS equation

In this section, we present the dissipative (2+1) dimensional AKNS equation [29], $u = u(x, y, t)$ and $R_x \times R_y \times R_t \to R$,

$$4u_{xt} + u_{xxxy} + 8u_{xy}u_x + 4u_{xx}u_y + \alpha u_{xx} = 0, \tag{2.1}$$

where α is a constant, and the coefficient $\alpha \neq 0$, shows that the system has dissipative effect. Some solutions of Eq. (2.1) have been obtained by using multidimensional Riemann theta function, simplified form of the bilinear method, further improved extended homoclinic test approach (EHTA) [29–31].

Let $y = x$, $\alpha = 0$, then (2.1) can be reduced to the Ablowitz–Kaup–Newell–Segur (AKNS) equation. Eq. (2.1) has been studied in detail by many researchers and obtained various exact solutions which include hyperbolic functions, the trigonometric functions and the rational functions. For example Liu applied the Bell polynomials to study the integrability [32], like the BT and the Lax pair. Bruzon et al. studied the AKNS equation and derived some explicit solutions using the classical Lie symmetry