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A variational model for multiscale texture extraction

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ABSTRACT

We propose a hierarchical (BV, G) variational decomposition model for multiscale texture extraction in this paper, which can offers a hierarchical, separated representation of image texture in different scales. The proposed hierarchical decomposition is obtained by replacing the fixed scale parameter of the A^2BC model with a varying sequence. Some properties of this hierarchical decomposition are presented and its convergence is proved. We adopt Euclidean projection algorithm to solve this hierarchical decomposition model numerically. In addition, we use this hierarchical decomposition to achieve the multiscale texture extraction. The performance of the proposed model is demonstrated with both synthetic and real images.

1. Introduction

Texture is a primary visual cue for pattern recognition and is connected with the human visual perception of coarseness or smoothness of image features. Texture gives us information about the spatial arrangement of color or intensities in an image or selected region of an image. When it is defined in a quantitative sense, texture is a property that relates to the nature of the periodic variability of pixel values. A visually smoother texture would contain only slight changes in digital number (DN) values over an area, while a visually coarse texture would contain many abrupt changes in DN values over an area.

Texture analysis has a wide range of applications such as remote sensing, medical diagnosis, document analysis, target detection, image segmentation, image classification, and so on. Of all these applications of texture analysis, texture extraction may be the most important preliminary work. By far there are many different methods used to extract textural information from images, which can be categorized into four major classes [1,2]: characterized as statistical (e.g. [3,4]), structural (e.g. [5,6]), model-based (e.g. [7–13]) and frequency-based (e.g. [14–23]).

Recently, an image decomposition method based on variational theory has received more and more attention and has been studied by many researchers (e.g. [24–38]). These decomposition methods can actually be used to achieve the texture extraction. Given an image function $f(x,y):\ \Omega \in \mathbb{R}^2 \to \mathbb{R}$ where Ω is an open, bounded and connected subset, the variational decomposition of f, $f = u + v$, can be obtained by minimizing an energy functional, in which u represents cartoon or structure component of f, while v represents oscillatory component consisting of scale repeated detail, i.e. texture, which is what we need. We here give some classical examples of image decomposition using functional minimization, which are most related to our present work.

A celebrated decomposition is obtained by the total variation (TV) minimization model by Rudin, Osher and Fatemi (ROF) [34] for image denoising, in which an image $f \in L^2\left(\Omega\right)$ is split into $u \in \mathrm{BV}(\Omega)$ and $v \in L^2\left(\Omega\right)$:

$$(u,v) = arg\inf\left\{|u|_{\mathrm{BV}(\Omega)} + \mu\|v\|_{L^2(\Omega)},\quad f = u + v\right\}, \tag{1}$$

which yields the so-called $\left(\mathrm{BV}, L^2\right)$ decomposition. Here, $\mu > 0$ is a tuning parameter, and $|u|_{\mathrm{BV}(\Omega)}$ is the BV-seminorm of u, defined by

$$|u|_{\mathrm{BV}(\Omega)} = \int_\Omega |\nabla u| = \sup\left\{\int_\Omega u\mathrm{div}(\xi):\quad \xi \in C_c^1\left(\Omega;\quad \mathbb{R}^2\right),\ \left\|\xi\right\|_{L^\infty(\Omega)} \le 1\right\},$$

where ∇u is the generalized derivative of u, $\left\|\xi\right\|_{L^\infty(\Omega)} = \mathrm{ess}\ \sup\sqrt{\xi_1^2+\xi_2^2}$ with $\xi = (\xi_1, \xi_2)$. Model (1) is convex and easy to solve in practice. In addition, the function $u \in \mathrm{BV}\left(\Omega\right)$ allows for discontinuities along curves, thus edges and contours can be preserved in the restored image u.

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However model (1) has some limitations. For instance, the structure and BV pieces of f are often sent to the component v for any μ [31,32,37], so it does not represent well oscillatory details (such as texture) since an oscillatory function doesn't have small L^2-norm. Meyer [32] suggested using a weaker norm instead of L^2-norm for the oscillatory component, and this can be done by using generalized function. One of his choices is to use the dual norm of BV-seminorm for the oscillatory components. However, there is no known integral representation of a continuous linear functional on $\mathrm{BV}\left(\Omega\right)$. To address this problem, Meyer [32] used a slightly larger space $G\left(\Omega\right)=W^{-1,\infty}\left(\Omega\right)$ to approximate the dual of $\mathrm{BV}\left(\Omega\right)$.

Definition 1. $G\left(\Omega\right)$ consists of distributions v which can be written as

$$v=\partial_x g_1+\partial_y g_2=\mathrm{div}\left(g\right),\quad g\in L^{\infty}\left(\Omega;\mathbb{R}^2\right)$$

endowed with the norm

$$\|v\|_{G(\Omega)}=\inf\left\{\|g\|_{L^{\infty}(\Omega)}:\quad v=\mathrm{div}(g),\quad g\in L^{\infty}\left(\Omega;\mathbb{R}^2\right)\right\},$$

where $\|g\|_{L^{\infty}(\Omega)}=\mathrm{ess}\quad\sup\quad\sqrt{g_1^2+g_2^2}$.

A function belonging to $G\left(\Omega\right)$ may have large oscillations and nevertheless have a small G-norm. Thus the norm on $G\left(\Omega\right)$ is well-adapted to capture the oscillations of a function in an energy minimization method. Using $G\left(\Omega\right)$ to model oscillatory component v, Meyer [32] introduced the following (BV, G) variational decomposition model:

$$\inf_{u\in \mathrm{BV}(\Omega),v\in G(\Omega)}\left\{|u|_{\mathrm{BV}(\Omega)}+\mu\|v\|_{G(\Omega)},\quad f=u+v\right\}. \tag{2}$$

In theory, this decomposition model can better extract texture. However, it cannot be directly solved in practice due to the nature of the G-norm [24,25,33,37,38], for which there is no standard calculation of the associated Euler–Lagrange equation. Vese and Osher [37,38] first overcame this difficulty by replacing the space $G\left(\Omega\right)$ with $G_p\left(\Omega\right)=W^{-1,p}\left(\Omega\right)$ with $1\le p<+\infty$. Then, the (BV, G) decomposition model (2) is approximated by the following minimization problem:

$$\inf_{u\in \mathrm{BV}(\Omega),v\in G_p(\Omega)}\left\{|u|_{\mathrm{BV}(\Omega)}+\lambda\left\|f-u-v\right\|^2_{L^2(\Omega)}+\mu\|v\|_{G_p(\Omega)}\right\}, \tag{3}$$

where $\lambda,\ \mu>0$ are tuning parameters, $\|v\|_{G_p(\Omega)}=\inf\left\|\sqrt{g_1^2+g_2^2}\right\|_{L^p(\Omega)}$ with $v=\mathrm{div}(g_1,g_2)$. The first term insures that $u\in\mathrm{BV}\left(\Omega\right)$, the second gives that $f\approx u+v$, and the third term is a penalty on the norm in $G_p\left(\Omega\right)$ of v. Clearly, if $\lambda\to\infty$ and $p\to\infty$, this model is formally an approximation of the (BV, G) model (2) proposed originally by Meyer. In this decomposition, the image f is discomposed into three components, $f=u+\ v+r$ with $u\in\mathrm{BV}\left(\Omega\right)$, $v\in G_p\left(\Omega\right)$ and $r\in L^2\left(\Omega\right)$.

Later on, Osher, Solé and Vese [33] proposed a simplified approximated method corresponding to the case $p=2$ in (3). And then, the functional space $G\left(\Omega\right)$ is replaced by the dual space $H^{-1}(\Omega)$ of $H_0^1(\Omega)$. The decomposition model is defined as

$$\inf_{u\in \mathrm{BV}(\Omega),v\in H^{-1}(\Omega)}\left\{|u|_{\mathrm{BV}(\Omega)}+\lambda\left\|f-u\right\|^2_{H^{-1}(\Omega)}\right\}, \tag{4}$$

where $\left\|f-u\right\|_{H^{-1}(\Omega)}=\left\|\nabla\Delta^{-1}(f-u)\right\|_{L^2(\Omega)}$. This minimization problem has been solved using a fourth-order non-linear partial differential equation in [33], and gives a 2-tuple decomposition, $f=u+\ v$, such that $u\in\mathrm{BV}\left(\Omega\right)$ and $v\in H^{-1}\left(\Omega\right)$.

Aujol, Aubert, Blanc-Féraud and Chambolle (A^2BC) [25,26] studied model (3) and (4), and pointed out that these two models are both not the best approximations to Meyer's (BV, G) decomposition theoretically. They proposed another approach to approximate it, stated as the following minimization problem:

$$\inf_{u\in \mathrm{BV}(\Omega),v\in G_\mu(\Omega)}\left\{J\left(u\right)+\lambda\left\|f-u-v\right\|^2_{L^2(\Omega)}+J^*\left(\frac{v}{\mu}\right)\right\}, \tag{5}$$

where $\lambda,\ \mu>0$ are tuning parameters, $G_\mu\left(\Omega\right)=\left\{v\in G\left(\Omega\right):\ \|v\|_{G(\Omega)}\le\mu\right\}$, $J(u)=|u|_{\mathrm{BV}(\Omega)}$ and $J^*(v)$ is the Legendre–Fenchel transform of $J(u)$. Since $J(u)$ is one homogeneous (i.e. $J(\lambda u)=\lambda J(u)$ for $u\in\mathrm{BV}\left(\Omega\right)$ and $\lambda>0$), $J^*(v)$ is actually the indicator function of the closed convex set $G_1=\left\{v:\|v\|_{G(\Omega)}\le 1\right\}$, i.e.,

$$J^*(v)=\sup_{u\in \mathrm{BV}(\Omega)}\left\{\langle u,v\rangle-J(u)\right\}=\chi_{G_1}(v)=\begin{cases}0 & \text{if}\ \ v\in G_1\\ +\infty & \text{otherwise}\end{cases}.$$

Model (5) can be solved by the projection algorithm in the dual framework proposed by Chambolle [39]. In addition, Aujol et al. state that as $\lambda\to+\infty$, then model (5) coincides with Meyer's (BV, G) model.

Here, we would like to list some other related works on solving (BV, G) decomposition numerically. Le and Vese [31] introduced the Dirac function in (3) to compute the G_1-norm of v. Aujol and Chambolle [40] used projection algorithm and dichotomy to computer G-norm. Weiss, Aubert and Blanc-Féraud [41] proposed an efficient algorithm based on Nesterov scheme [42] for TV minimization, which was used to solve (BV, G) model.

The models mentioned above are examples for a larger class of variational decompositions with fixed scales; the scale parameters in these models are fixed. If they are used for texture extraction, one can only achieve a fixed scale texture extraction. It has been argued that a human visualizes a scene in multiple scales [43,44]. The multiscale approaches (e.g. [43–48]) are appropriate for texture analysis because a single scale may be not a perfect simulation of the human visual perception on texture elements. In order to achieve reliable texture information in different scales, large-scale and small-scale behaviors should both be investigated and incorporated appropriately. Thus, a natural way to address this problem is the multiscale analysis. Frequency-based methods such as Gabor filter [15–17] and wavelet transform [18–20], trying to characterize texture through filter responses directly, can produce a good multiscale texture extraction. These two multiscale techniques transforming images into a hierarchical representation can achieve a good simulation of the human visual perception on texture elements.

In [46], we proposed a multiscale texture extraction method based on variational decomposition, where the $\left(\mathrm{BV}, G_p\right)$ decomposition model (3) proposed by Vese and Osher [37,38] is used to extract texture. It is known that the space of G_p is only a rough approximation of G proposed originally by Meyer. So in this paper, we propose a hierarchical (BV, G) variational decomposition based on the A^2BC model, and then we use it to achieve multiscale texture extraction. We here adopt the A^2BC model because it is the best approximation to Meyer's (BV, G) decomposition theoretically. In our hierarchical decomposition, the scale parameter μ in the A^2BC model, used to measure the texture, is not fixed, but varies over a sequence of binary scales. So, this hierarchical decomposition enables us to capture successively the oscillation of f, which lies in the intermediate scale spaces between $L^2\left(\Omega\right)$ and $G\left(\Omega\right)$. Then, the extracted texture from f is not predetermined, but resolved in terms of layers of intermediate scales.

It should be pointed out that Tadmor et al. have proposed the hierarchical $\left(\mathrm{BV}, L^2\right)$ decomposition (see [36,49]) and the