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A derivative method of phase retrieval based on two interferograms with an unknown phase shift

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ABSTRACT

Phase retrieval is one of the important study fields in the optical metrology of phase objects. In this paper, a rapid derivative method of phase retrieval is proposed that can both extract phase and determine an unknown phase shift based on two-step phase-shifting interferometry (PSI). By numerically calculating the first-order derivatives of two PSI images with the random phase distribution, one can directly obtain the quantitative phase image, without the need for dealing with the background term and reference intensity. We give the method with the theory, and illustrate it using the simulations of a ball and a red blood cell (RBC) and an optical experiment of a polystyrene sphere. From which, the effectiveness and accuracy of this method is verified, but the error of the experimental result is larger than that of the simulated result.

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1. Introduction

Phase-shifting interferometry (PSI) has become an important technique in optical metrology for measuring the phase from interferograms [1,2]. In recent years, many achievements related to PSI have been presented, especially phase retrieval and phase shift extraction [3–10]. In standard PSI, to retrieval the measured phase, a sequence of N-step phase-shifting interferograms ($N \geq 3$) with the known phase shift are required. Usually the phase shift between two adjacent steps is a special constant. In fact, it is a very strict requirement, which is often difficult to meet exactly in real cases. In order to settle this problem, many generalized methods with unknown phase shifts have been proposed, such as advanced iterative algorithm (AIA) [6], two normalized difference maps algorithm [7] and the blind self-calibrating algorithm [8]. These methods need three or more frames and hence which are not feasible for real-time measurement.

Naturally the method using few interferograms in PSI can simplify the computation process and enhance the retrieval efficiency. Recently, some two-step phase retrieval algorithms with an unknown phase shift have been reported [11–19]. For example, based on the statistical nature that the phase distribution of the object wave is random, Cai et al. [11,12] derived some different kinds of algorithms to compute the phase shift and then retrieve the information related to the object wave. But it is required to know the reference wave intensity or measure it separately. In Ref. [13], by searching the extreme value of the interference term, an accurate and rapid two-step phase demodulation algorithm is proposed. In Ref. [14], Vargas proposed a different phase retrieval method based on the Gram–Schmidt orthonormalization algorithm. However, in above these two methods, the background term is required to be eliminated in advance, which is closely related to the accuracy of phase reconstruction. In Ref. [19], the Hilbert transform is applied to phase retrieval, in which the background is eliminated by the subtraction operation, but it is required to measure the phase shift separately in advance.

In this study, considering a common assumption for phase objects that both the background and modulation intensities vary much slower than the measured phase [20], we present a derivative method for phase reconstruction under two-step PSI with an unknown phase shift. The phase shift can be determined by a simple formula which is derived by the ratio of the first-order derivatives of two interferograms, and then the phase can be retrieved.

2. Method

In two-step off-axis PSI, the intensity distributions of the interferograms recorded before and after introducing a phase shift of α can be described as:

$$I_1(x, y) = I_0(x, y) + \gamma(x, y)\cos[\varphi(x, y) + kx], \quad (1)$$

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and

$$I_2(x,y) = I_0(x,y) + \gamma(x,y)\cos[\varphi(x,y) + kx + \alpha], \quad (2)$$

where x and y denote spatial coordinates, $I_0(x,y)$, $\gamma(x,y)$ and $\varphi(x,y)$ are the background intensity, the modulation amplitude, and the measured phase associated with the sample, respectively. k is the spatial carrier frequency along x direction, which is determined by the tilt angle between the sample and reference beams. The phase shift, α, is supposed in the$(0,\pi)$ range to avoid possible ambiguity. For most phase objects, such as biological cell, both the background term $I_0(x,y)$ and the modulation factor $\gamma(x,y)$ are constant over the interferogram approximately [21], thus the following approximation is satisfied,

$$\frac{\partial I_0(x,y)}{\partial x} \approx 0, \quad \frac{\partial \gamma(x,y)}{\partial x} \approx 0 \quad (3)$$

So, according to Eq. (3), the first-order derivatives of two interferograms with respect to x can be expressed as

$$I_1^{(1)} = \gamma(x,y)\left\{-\sin[\varphi(x,y) + kx]\right\}\left[\frac{\partial\varphi(x,y)}{\partial x} + k\right], \quad (4)$$

and

$$I_2^{(1)} = \gamma(x,y)\left\{-\sin[\varphi(x,y) + kx + \alpha]\right\}\left[\frac{\partial\varphi(x,y)}{\partial x} + k\right], \quad (5)$$

respectively. When the phase shift α is known, the phase $\varphi(x,y)$ can be directly computed with the above two equations, which is given by

$$\varphi(x,y) = \arctan\left[\frac{{I_1}^{(1)}\sin\alpha}{{I_2}^{(1)} - {I_1}^{(1)}\cos\alpha}\right] - kx. \quad (6)$$

However, it is often required to measure the extract phase shift α in many real experiments. Because the actual phase shift introduced by a phase shifter is more or less different from its preset value. For settling the problem, many algorithms of phase shift extraction have been proposed. In this work, we try to use the statistical average method for the purpose. To explain the principle of this method, let us calculate the ratio of Eqs. (5) and (4). We then have

$$\frac{{I_2}^{(1)}}{{I_1}^{(1)}} = \frac{\sin[\varphi + kx + \alpha]}{\sin(\varphi + kx)} = \cos(\alpha) + \cot(\varphi + kx)\sin(\alpha). \quad (7)$$

By calculating the average of Eq. (7), we have

$$\left\langle \frac{{I_2}^{(1)}}{{I_1}^{(1)}} \right\rangle = \left\langle \cos(\alpha) \right\rangle + \left\langle \cot(\varphi + kx)\sin(\alpha) \right\rangle = \cos(\alpha) + \left\langle \cot(\varphi + kx) \right\rangle \sin(\alpha), \quad (8)$$

where $\langle\rangle$ is the average over all pixels. Because an oblique incident plane wave is used as a reference wave, a linear phase factor uniformly distributed between 0 and 2π, is sufficiently generated in the recorded plane. Thus, the phase randomness condition in the recorded plane is satisfied [22]. Consequently, the second term of Eq. (8) is reduced to zero. Therefore, Eq. (8) can be simplified as:

$$\left\langle \frac{{I_2}^{(1)}}{{I_1}^{(1)}} \right\rangle = \cos(\alpha). \quad (9)$$

Obviously, the phase shift α can be calculated using the above equation, and has a unique value in the range of $(0,\pi)$. It indicates that the unknown phase shift can be calculated using the simple average operation. After determining the phase shift, the phase can be retrieved conveniently using Eq. (6).

3. Simulation

In order to verify the effectiveness of our proposed method, a series of computer simulations have been done. In the first simulation, a ball with the diameter of 8 μm has been used as the object. Its refractive index is set to be 1.59 and that of the surrounding medium is set to be1.47, which they are the same as the refractive indices of a polystyrene sphere and the glycerol as the surrounding medium, respectively. In our simulation, the other parameters are as follows: the wavelength of the laser is 632.8 nm, both the sample and the reference waves are assumed as plane waves, and the spatial carrier frequency, k is 0.1359 rad/pixel. In addition, the phase shift was preset as $\pi/2$. According to the above parameters, two phase-shifted interferograms (1024 × 1024 pixels) can be obtained with Eqs. (1) and (2), respectively, as shown in Fig. 1(a) and (b). Fig. 1(c) and (d) are the first-order derivatives of Fig. 1(a) and (b), respectively. Then, we can directly calculate the phase shift from these two derivatives with Eq. (9). The calculated result is 1.5726 rad and the deviation is 0.0018 rad compared to the preset value. After determining the phase shift, the phase can be reconstructed according to Eq. (6), as shown in Fig. 1(e) and (f),

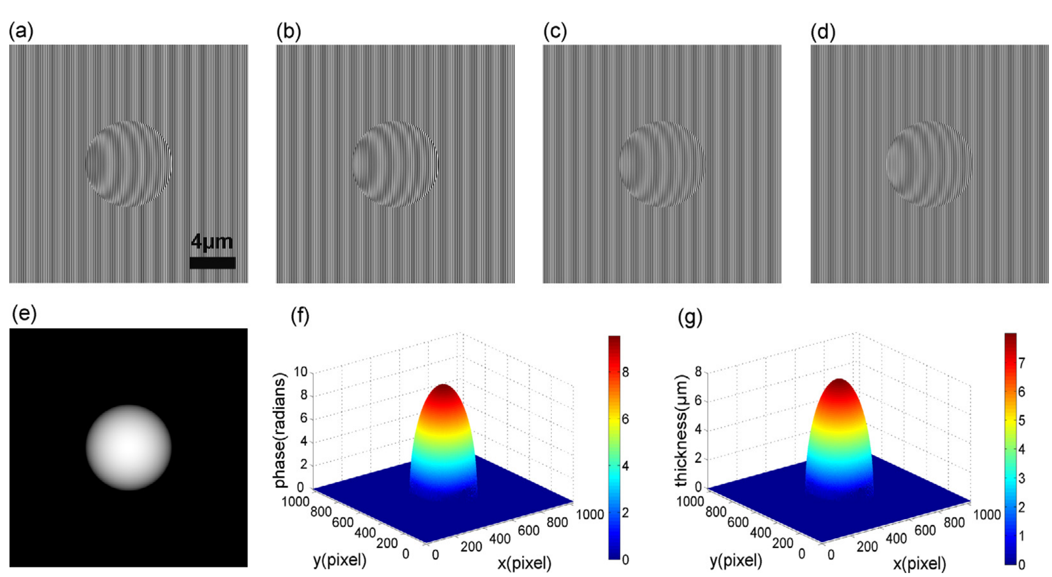

Fig. 1. Derivative method for phase reconstruction of a ball: (a) and (b) are two phase-shifted interferograms, (c) and (d) are the first-order derivatives of (a) and (b), respectively, (e) and (f) are the 2D display and 3D display of the reconstructed phase, (g) the thickness distribution.

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