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Transfer function of optical waveguide ring resonator in frequency domain for micro-optic gyro

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ABSTRACT

Resonator micro optic gyro (R-MOG) is a high accuracy inertial rotation sensor based on the Sagnac effect. Optical waveguide ring resonator is the core-sensing element in R-MOG. According to the feature of the input signal, we established the transfer function of a Si-based ring resonator model in frequency domain for R-MOG with the coupled-mode theory. According to using the derived transfer function in the frequency domain, we analyzed the demodulation characteristics of R-MOG using Bessel function expansion based on laser frequency modulation spectrum technique. And we analyzed the relationship between the frequency modulation demodulation output signal and the resonant frequency deviation and obtained the best modulation coefficient applied in laser PZT drive. When the modulation coefficient $M=2$, the maximum linear interval slope, the demodulation curve is the best.

1. Introduction

Resonant micro-optic gyro (R-MOG), which has excellent advantages such as high reliability, wide dynamic range, small size, and low cost, is one of the most promising candidates for the next generation inertial rotation sensors [1–3]. Angular velocity can be obtained in R-MOG by sensing the resonant frequency difference between the counterpropagating light beams transmitting in the waveguide ring resonator. In R-MOG system, the signal modulation and the detection technique are very important in the MOG system, because the detection precision decides the ultimate sensitivity of the gyro directly. Several groups have researched and proposed different R-MOG schemes [4–8]. There are two main open-loop detection methods: the laser frequency modulation (FM) [9,10] and phase modulation (PM) [11–13]. In the PM spectroscopy technique, the modulation signal and the feedback signal are separated. But the modulation signal and the feedback signal is simultaneously applied to the laser in the FM method. On the other hand, LiNbO3 phase modulators were adopted to realize the frequency modulation, which made it incompatible with silicon process technology and standard CMOS electronics, and thus it is harder to integrate into complete systems on a chip [14]. And because the frequency of the lightwave emitted from the fiber laser can easily be tuned by PZT directly. Due to the miniaturization, integration is the development trend of resonator integrated optical gyro, the ultimate goal is to integrated all complements including laser, acousto-optic frequency shifter, resonator into a silicon chip. So laser frequency modulation detection integrated on silicon based optical waveguide is a relatively ideal detection technology. In this paper, we reported on the transfer function of a Si-based ring resonator model in frequency domain analyzed the relationship between the frequency modulation demodulation output signal and the resonant frequency deviation. Result: when the modulation coefficient $M=2$, the maximum linear interval slope, the demodulation curve is the best.

2. Theoretical frameworks

Fig. 1 shows the open-loop detection setup of the R-MOG by using the FM spectroscopy technique. The coupling ratios for couplers C1, C2, and C3 are all designed as 50%. The output light from the fiber laser (FL) is split into two beams by C1. The source was modulated by PZT before being injected into the resonator. Subsequently, the signals from PD1 and PD2 are demodulated by lock-in amplifiers LIA1 and LIA2. In the CCW direction, the frequency of the FL (f_{FL}) is locked to the CCW resonant frequency f_{CCW} by the feedback circuit (FBC). When the device is rotating, the acousto-optic frequency shifter (AOFS) controls the resonant frequency difference in the CW and CCW. Measuring the frequency difference between the imposed frequency shifts demonstrates a measurement of the rotation rate. In the open-loop operation, the output signal from LIA2 will give out the gyro output.

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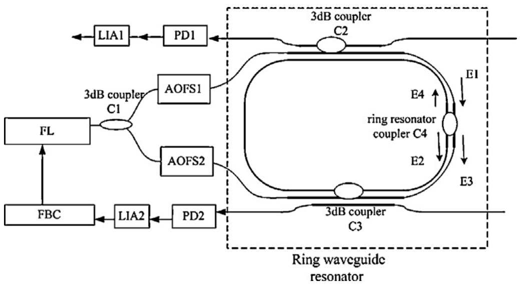

Fig. 1. Schematic diagram of ring resonator.

Optical waveguide ring resonator is the key sensing unit in R-MOG system; we first discuss the resonance characteristics at ideal source. Namely not thinking of the influence of spectral linewidth of laser on resonance characteristics in ring resonator. The directional coupler of ring resonator has 4 ports.

In order to derive the expression for the power transmission of such a resonator, assume that the couplers are lossless and have a frequency independent field coupling coefficient k. When the optical wave at port 4 has circulated one turn in the ring waveguide at steady-state resonance.

The electric field amplitude at port 2 can be written as

$$E_2 = e^{-\alpha L} \cdot e^{j\beta L} E_4 \tag{1}$$

where $E_i (i = 1, 2, 3, 4)$ is electric field amplitude at port i, α is the ring waveguide loss per unit length, and L is the ring waveguide length, β is the optical propagation constant.

By coupled-mode theory [14,15], the output electric field complex amplitudes in the coupler can be expressed as

$$E_3 = (1-\gamma_0)^{1/2}[(1-k)^{1/2}E_1 + jk^{1/2}E_2] \tag{2}$$

$$E_4 = (1-\gamma_0)^{1/2}[(1-k)^{1/2}E_2 + jk^{1/2}E_1] \tag{3}$$

$$\left|\frac{E_3}{E_1}\right|^2 = (1-\gamma_0)\left[1 - \frac{k(1-k-A)/(1-k)}{1+A-2A^{1/2}\cos\ \beta \cdot L}\right] \tag{4}$$

$$\left|\frac{E_4}{E_1}\right|^2 = \frac{(1-\gamma_0)k}{1+A-2A^{1/2}\cos\ \beta L} \tag{5}$$

$$A = (1-k)(1-\gamma_0)e^{-2\alpha L} \tag{6}$$

where k is the intensity coupling coefficient, γ is the coupler insertion loss.

At optimum resonance, the output energy at port 3 should be zero. Thus the phase condition and the optimum resonance amplitude condition can be respectively obtained

$$\beta L = p \cdot 2\pi \tag{7}$$

$$k = k_r, \quad \text{where} \quad k_r = 1-(1-\gamma_0)e^{-2\alpha L} \tag{8}$$

The phase changes in the vicinity of resonance at ports 3 can be obtained from Eqs. (1)–(3) by setting $k = k_r$. The relative phases of output port are

$$\phi_{31} = -\tan^{-1}\left[\frac{k_r \sin\ \beta L}{(2-k_r)(1-\cos\ \beta L)}\right] \tag{9}$$

Ring resonator have two important performance parameters [14], the finesse and the free spectral range (FSR). The finesse of F defined as the ratio of FSR to the full frequency width at half maximum FWHM, FWHM = $2\pi\Delta\omega_H$, that is [15],

$$F = \frac{\text{FSR}}{\text{FWHM}} = \frac{\pi}{2\sin^{-1}[k_r/(2(1-k_r)^{1/2})]} \tag{10}$$

$$\text{FSR} = \frac{c}{n_1 L} \tag{11}$$

where L is the length the ring, c is the light velocity in a vacuum and n_1 is the index of refraction.

The input and output of transfer function of the resonator can be written in the form

$$H_R(\omega) = h_R(\omega)\exp[j\phi(\omega)] \tag{12}$$

where $h_R(\omega)$ is the amplitude of the transfer function of the resonator, $\phi(\omega)$ is the phase delay section.

(1) When the optimum resonance amplitude condition is satisfied, output intensities to the input intensity of ring resonator is:

$$\left|\frac{E_3}{E_1}\right|^2 = \frac{2(1-\gamma_0)(1-k_r)(1-\cos\ \beta L)}{1+(1-k_r)^2-2(1-k_r)\cos\ \beta L} \tag{13}$$

When the phase condition is not is satisfied, namely $\beta L \neq p{\cdot}2\pi$,we can expressed:

$$\begin{cases} \dfrac{\beta L}{2\pi} = p + \Delta x \\ |\Delta x| \le \dfrac{1}{2} \end{cases} \tag{14}$$

Then let $\Delta x = \frac{\omega-\omega_R}{2\pi \text{FSR}}$

ω_R is the resonant angular frequency of the resonator, ω is the input light angular frequency of the resonator. $\omega = 2\pi f$, $\cos\beta L$ is written in the form of half-width function formula in Eq. (13)

$$\sin\frac{\beta L}{2} = \sin\frac{(2\pi p + 2\pi\Delta x)}{2} = \sin\pi\left(\frac{\omega-\omega_R}{2\pi \text{FSR}}\right) = \sin\frac{\omega-\omega_R}{2\text{FSR}} \tag{15}$$

Since ω and ω_R are very close, we can get:

$$\sin\frac{\omega-\omega_R}{2\text{FSR}} \approx \frac{\omega-\omega_R}{2\text{FSR}} = \frac{nL}{2c}(\omega-\omega_R) \tag{16}$$

And because of $F = \pi/(2\sin^{-1}[k_r/(2(1-k_r)^{1/2})])$ and $\Delta\omega_H = (2\pi\text{FSR})/F$, We can get:

$$\Delta\omega_H = \frac{4c}{nL}\sin^{-1}\left[\frac{k_r}{2(1-k_r)^{1/2}}\right] \tag{17}$$

Put formula (15)–(17) into (13), we can get:

$$\left|\frac{E_3}{E_1}\right|^2 = \frac{(1-\gamma_0)(\omega-\omega_R)^2}{(\Delta\omega_H/2)^2+(\omega-\omega_R)^2} \tag{18}$$

Assuming $\gamma_0 = 0$, therefore, the magnitude of the transfer function of the resonator can be written in the following form:

$$h_R(\omega) = \sqrt{\frac{(\omega-\omega_R)^2}{(\Delta\omega_H/2)^2+(\omega-\omega_R)^2}} \tag{19}$$

(2) When the optimal amplitude and phase resonance condition is not satisfied, By the (4) equation, $|E_3/E_1|^2 \neq 0$. Let $\eta = |E_3/E_1|^2_{\min}$, $\rho = 1-\eta$ (ρ is called the resonance depth of the resonator), Then $|E_3/E_1|^2$ can be written as:

When $\cos\beta L = 1$, there

$$\left|\frac{E_3}{E_1}\right|^2 = 1 - \frac{k(1-k-A)/(1-k)}{1+A-2A^{1/2}} = 1 - \frac{k(1-e^{-2\alpha L})}{[1-(1-k)^{1/2}e^{-\alpha L}]^2} \tag{20}$$

when $\cos\beta L \neq 1$, there

$$\left|\frac{E_3}{E_1}\right|^2 = 1 - \frac{k(1-e^{-2\alpha L})}{[1-(1-k)^{1/2}e^{-\alpha L}]^2 + 4(1-k)^{1/2}e^{-\alpha L}\sin^2(\beta L/2)} \tag{21}$$