Optik 126 (2015) 2199–2207

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Dynamics for the collective model of two atoms interacting with two-mode quantized radiation fields in a Raman type process

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ARTICLE INFO

Article history:
Received 13 April 2014
Accepted 23 May 2015

Keywords:
Jaynes–Cummings (J–C) model
Two-mode Raman model
Dicke-type atomic states
Atomic population dynamics
Photon statistics

PACS:
42.50 Hz
42.50. Dv
42.50. Ct

ABSTRACT

We obtain exact solutions for the collective model of two identical two-level atoms interacting with two quantized cavity electromagnetic fields in a Raman type process. The unitary transformation method that is used to solve the time dependent problem, also gives the eigen solutions of the interacting Hamiltonian at the same time. We study the dynamics of atomic population and photon statistics in the two cavity modes. We obtain evidence of co-operative effects in antibunching and anticorrelations in the modes.

1. Introduction

The properties of non-linear interaction models are of great interest in quantum optics because of their applications in the field of two-mode two-photon micromasers [1,2]. Encouraged by the experimental realization of the two-photon one atom micromaser on Rydberg transitions in a microcavity [3], a number of schemes for a two mode two-photon maser with a Rydberg atom in a microwave cavity have been proposed and analyzed [4–6].

Non linear processes involving two-photon transitions are important in quantum optics due to high degree of correlation among the emitted photons. Two-photon transitions play an important role in the production of non-classical light and have been utilized to produce squeezed light [7,8]. At present we find a number of publications on single atom two-mode two-photon processes whereas the publications concerning the many/two atom non-degenerate processes are still few in number [9–11]. This is the motivation behind the study of the collective dynamics of a system of two identical two-level atoms that resonantly interact with two quantized modes of electromagnetic fields via Raman type process.

In the present paper we study the quantum dynamics of the Two-atom Raman coupled model interacting with two quantized cavity fields in an ideal cavity. Comparison of the results with the corresponding data obtained for the one atom model [4,5] makes it possible to yield the collective features of the model under study.

For obtaining the eigenfunctions and eigenvalues of the Hamiltonian of the interacting system a straightforward method, that of the unitary transformation in quantum mechanics by Sudha Singh [12] has been used. The method besides being more general is mathematically simpler.

The paper is organized in the following way. In Sect. 2 we present Two-atom Two-mode Raman Model describing the relevant Dicke-type [13] atomic states. Explicit expressions for the eigen functions and eigen values of the Hamiltonian of the interacting system has been obtained. In Sect. 3 we study the atomic dynamics of the two-atom system and its behavior for different values of two photon detuning parameter. The time dependence of the mean number of photons in the mode has been analyzed in Sect. 4. In sect. 5 we study the statistical properties of the field modes. We conclude the paper with a brief summary.

2. Two-atom two-mode Raman model

In dealing with two-mode Raman type processes, a three-level system of energies E_1, E_2 and E_3

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http://dx.doi.org/10.1016/j.ijleo.2015.05.107
0030-4026/

in the Λ configuration is considered to be interacting with a pump mode ω_1 and a Stoke mode ω_2 [4,14,15]. The Hamiltonian of the system is written as [4,15–17]

$$\hat{H} = \sum_{i=1}^{3} E_i \sigma_{ii} + \hbar\omega_1 \hat{a}_1^\dagger \hat{a}_1 + \hbar\omega_2 \hat{a}_2^\dagger \hat{a}_2 + \hbar g_1(\hat{a}_1 \hat{\sigma}_{31} + \hat{a}_1^\dagger \sigma_{13}) + \hbar g_2(\hat{a}_2 \hat{\sigma}_{32} + \hat{a}_2^\dagger \hat{\sigma}_{23}) \quad (1)$$

here the symbols $\hat{a}_j^\dagger$ ($j = 1, 2$) and $\hat{a}_j$ represent the field creation and annihilation operators for modes 1 and 2, $\hat{\sigma}_{ii} = |i\rangle\langle i|$ are the level occupation number and $\hat{\sigma}_{ij} = |i\rangle\langle j| \rightleftarrows (i \neq j)$ are the transition operators from level j to i. Levels 3 and 1 (2) are coupled by a dipole-coupling constant g_1 (g_2). There is no direct coupling between levels 1 and 2. The quantities Δ_1 and Δ_2 denote detuning given by $\hbar\Delta_j = (E_3 - E_j) - \hbar\omega_j, j = 1, 2$. It has been shown by Wu [17] that this three level problem can be exactly transformed into a two-level problem, regardless of whether the detuning is small or large. The corresponding two-level Hamiltonian under the rotating wave approximation (RWA) reads [4,5,14–17]

$$\hat{H} = \frac{1}{2}\hbar\omega_o \hat{\sigma}_3 + \hbar\omega_1 \hat{a}_1^\dagger \hat{a}_1 + \hbar\omega_2 \hat{a}_2^\dagger \hat{a}_2 + \hbar g\left(\hat{\sigma}_+ \hat{a}_2^\dagger \hat{a}_1 + \hat{\sigma}_- \hat{a}_1^\dagger \hat{a}_2\right) \quad (2)$$

The pump radiation mode ω_1 and the Stokes radiation mode ω_2 are in two mode resonance with the $|1\rangle$ to $|2\rangle$ transition such that $\hbar\omega_0 = (E_2 - E_1) = \hbar(\omega_1 - \omega_2)$. This Hamiltonian ignores Stark shifts due to coupling through virtual $|j\rangle$ levels. The $\hat{\sigma}'$s are 2×2 Pauli matrices.

The number states for the field modes are the direct product of number states for modes 1 and 2 i.e. $|n_1, n_2\rangle_F = |n_1\rangle_{F_1} \otimes |n_2\rangle_{F_2}$.

The many atom case is constructed just as for the usual Dicke model [9,13]. We define the collective atomic operators $\hat{R}_1, \hat{R}_2$, and$\hat{R}_3$ as

$$\hat{R}_k = 1/2\sum_{j=1}^{N} \hat{\sigma}_k^j, \quad \hat{R}_\pm = \sum_{j=1}^{N} \hat{\sigma}_\pm^j \quad (3)$$

where N is the number of atoms, $k = 1, 2, 3$ and $\hat{\sigma}_\pm^j$ are the atomic operators for the jth atom.

The operators $\hat{R}_\pm$, $\hat{R}_k$ and $\hat{R}^2 = \hat{R}_1^2 + \hat{R}_2^2 + \hat{R}_3^2$ obey the commutation relations for general angular momentum operators. The Hamiltonian for the N-atom case is then obtained as

$$\hat{H} = \hbar\omega_o \hat{R}_3 + \hbar\omega_1 \hat{a}_1^\dagger \hat{a}_1 + \hbar\omega_2 \hat{a}_2^\dagger \hat{a}_2 + \hbar g\left(\hat{R}_+ \hat{a}_2^\dagger \hat{a}_1 + \hat{R}_- \hat{a}_1^\dagger \hat{a}_2\right) \quad (4)$$

or,

$$\hat{H} = \hbar\omega_o \hat{R}_3 + \hbar\omega_1 \hat{a}_1^\dagger \hat{a}_1 + \hbar\omega_2 \hat{a}_2^\dagger \hat{a}_2 + \hbar g(\hat{P}_+ + \hat{P}_-) \quad (5)$$

with

$$\hat{P}_+ = \hat{R}_+ \hat{a}_2^\dagger \hat{a}_1; \quad \hat{P}_- = \hat{R}_- \hat{a}_1^\dagger \hat{a}_2 \quad (6)$$

We consider only the special case for $N = 2$. The basis vectors for the atomic system can be represented as $|\psi\rangle = |m_h\rangle$ where m_h stands for number of atoms occupying their higher energy states. Since the different atom operators commute with each other, the following operations hold

$$\begin{aligned} \hat{R}_3 |m_h\rangle &= (m_h - N/2)|m_h\rangle \\ \hat{R}_+ |m_h\rangle &= (N - m_h)|m_h + 1\rangle \\ \hat{R}_- |m_h\rangle &= m_h |m_h - 1\rangle \end{aligned} \quad (7)$$

For the two-atom case, the possible two-atom states are

$$\begin{aligned} |0\rangle &= |1\rangle_{A_1} \otimes |1\rangle_{A_2} = |1, 1\rangle_A. \\ |1\rangle &= \left[|1\rangle_{A_1} \otimes |2\rangle_{A_2} + |2\rangle_{A_1} \otimes |1\rangle_{A_2}\right] = \left[|1, 2\rangle_A + |2, 1\rangle_A\right] \\ |2\rangle &= |2\rangle_{A_1} \otimes |2\rangle_{A_2} = |2, 2\rangle_A \end{aligned} \quad (8)$$

where A_1 and A_2 refer to atom 1 and atom 2, respectively. We introduce the Dicke states $|j, \quad m\rangle_D$. For the two-atom system we have $j = 1$ with $m = 1, 0, -1$. These states are related to the two-atom states above according to

$$\begin{aligned} |1, -1\rangle_D &= |1, 1\rangle_A \\ |1, 0\rangle_D &= \frac{1}{\sqrt{2}}\left[|1, 2\rangle_A + |2, 1\rangle_A\right] \\ |1, +1\rangle_D &= |2, 2\rangle_A \end{aligned} \quad (9)$$

If we assume that initially both the atoms are in the ground state having n_1 and n_2 photons with frequencies ω_1 and ω_2, respectively then, at $t = 0$ we have

$$|\psi(0)\rangle = |0\rangle |n_1, n_2\rangle_F = |0, n_1, n_2\rangle = |1, -1\rangle_D |n_1, n_2\rangle \quad (10)$$

On the other hand, if we assume that initially both the modes are in coherent states, at $t = 0$ we have

$$\begin{aligned} |\psi(0)\rangle &= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} C_{n_1}(\alpha_1) C_{n_2}(\alpha_2) |0, n_1, n_2\rangle \\ &= |1, -1\rangle_D \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} C_{n_1}(\alpha_1) C_{n_2}(\alpha_2) |n_1, n_2\rangle \end{aligned} \quad (11)$$

where $|C_{n_i}(\alpha_i)|^2 = P_{n_i}(\bar{n}_i) = |\langle n_i | \alpha_i \rangle|^2 = \exp(-\bar{n}_i) \frac{\bar{n}_i^{n_i}}{n_i!} (i = 1, 2)$

where $\bar{n}_i$ = initial average number of photons in the ith mode. $P_{n_1}(\bar{n}_1)$ and $P_{n_2}(\bar{n}_2)$ represents the coherent field probability distribution functions for photon numbers in the Poisson statistics.

The general state vector for the system at any time t can be expressed as the linear combination of the basis eigenkets of the interacting system designated as

$$\begin{aligned} |\psi(t)\rangle = \sum_{n_1, n_2=0}^{\infty} &\left[A_{1,1}^{n_1, n_2}(t)|1, 1\rangle_A |n_1, n_2\rangle_F + A_{1,2}^{n_1, n_2}(t)\right. \\ &\times \left(|1, 2\rangle_A + |2, 1\rangle_A\right)|n_1 - 1, n_2 + 1\rangle_F \\ &\left. + A_{2,2}^{n_1, n_2}(t)|2, 2\rangle_A |n_1 - 2, n_2 + 2\rangle\right] \end{aligned} \quad (12)$$

$$\begin{aligned} |\psi(t)\rangle = \sum_{n_1, n_2=0}^{\infty} &\left[A_{1,1}^{n_1, n_2}(t)|0, n_1, n_2\rangle_F + A_{1,2}^{n_1, n_2}(t)|1, n_1 - 1, n_2 + 1\rangle\right. \\ &\left. + A_{2,2}^{n_1, n_2}(t)|2, n_1 - 2, n_2 + 2\rangle\right] \end{aligned} \quad (13)$$

In terms of the Dicke states of Eq. (9)

$$\begin{aligned} |\psi(t)\rangle = \sum_{n_1, n_2=0}^{\infty} &\left[A_-^{n_1, n_2}(t)|1, -1\rangle_D |n_1, n_2\rangle_F\right. \\ &+ \sqrt{2} A_0^{n_1, n_2}(t)|1, 0\rangle_D |n_1 - 1, n_2 + 1\rangle_F \\ &\left. + A_+^{n_1, n_2}(t)|1, +1\rangle_D |n_1 - 2, n_2 + 2\rangle_F\right] \end{aligned} \quad (14)$$

here the relabeling of the A co-efficients from Eqs. (13) and (14) is obvious.

The state vector $|\psi(t)\rangle$ develops from the state vector $|\psi(0)\rangle$ at $t = 0$ according to

$$|\psi(t)\rangle = \hat{T}(t)|\psi(0)\rangle \quad (15)$$