Optik 126 (2015) 1640–1644

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

A novel algorithm based on histogram processing of reliability for two-dimensional phase unwrapping

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ARTICLE INFO

Article history:
Received 13 March 2014
Accepted 29 April 2015

Keywords:
Phase unwrapping
Fringe analysis
Digital holography

ABSTRACT

A fast approach for two-dimensional phase unwrapping is presented. Reliability functions with fixed value range are defined for pixels and edges. Through histogram statistics for reliability values of edges, all edges are allocated to the corresponding subintervals of histogram. The proposed algorithm unwraps the phase subinterval by subinterval and for each subinterval edge by edge. A number of simulated and experimental results show that the proposed algorithm reacts satisfactorily to random noise and discontinuities in the wrapped phase distribution. The execution time of this algorithm is less than 60 ms for an image size of 800 × 800 pixels on a PC system generally. This algorithm can achieve quasi-real-time performance.

1. Introduction

Fringe analysis has been widely applied to many different fields, such as speckle and holographic interferometry, for measurement of many physical parameters, such as displacement and surface profile, which are calculated by means of the arctangent function and are encoded in the form of two-dimensional phase maps. These phases are given modulo 2π, caused by the mathematical function. And therefore, many 2π jumps appear in the resulting phase distributions. Unwrapping is the procedure by which many appropriate multiples of 2π are added to pixels to restore the original continuous phase distributions.

For a perfect wrapped phase map, phase unwrapping is a simple and fast procedure [1]. But real wrapped phase maps always contain many different issues which can be roughly categorized as high-frequency and high-amplitude noise, discontinuous phase jumps, and insufficient sampling in local areas. Many different algorithms have been developed to solve these problems for a long time [2], but requirements for accuracy and speed are not always met at the same time, especially for complex surface topography measurement. These algorithms can be generally grouped into three categories [3,4]: global algorithms, region algorithms, and path-following algorithms.

The global algorithms define global functions and determine the unwrapped phase values under the conditions where these functions have minimum values [5–10]. Most of these algorithms in this category are known to be robust but computationally intensive.

The region algorithms can be subclassified into two groups. (1) The region-based algorithms [11,12] divide the image into homogenous regions within which the phase is continuous. (2) The tile-based algorithms [3,13,14] partition the image into smaller areas geometrically. All these algorithms process each region individually to limit the propagation of errors, and then the processed regions are joined together to form larger regions until the whole image is processed. These algorithms provide a compromise between the robustness and the computational intensity [4].

The path-following algorithms can be subclassified into three groups. (1) Path-dependent algorithms [15] detect noisy points or discontinuities, and then select a best path to avoid the propagation of errors. These algorithms are generally fast but a correct result is not guaranteed if the phase map is corrupted by noise severely. (2) Residue-compensation algorithms [16–19] identify residue, then generate cut lines between positive and negative residues. By detouring residues and cut lines, the propagation of errors is avoided. These algorithms are also fast but not robust. (3) Quality guided path algorithms [4,20–24] determine the phase unwrapping path by using a quality map or reliability criterion. The unwrapping path, which follows a continuous or discontinuous path, starts from the highest-quality pixels and continues to the lower-quality ones until it finishes, for preventing error propagation. These algorithms are surprisingly robust in practice, but difficult to obtain real-time performance, due to a time-consuming sorting process for determining the unwrapping path. The proposed algorithm falls into this category fundamentally, but is more computationally efficient.

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http://dx.doi.org/10.1016/j.ijleo.2015.04.070

In this paper, the proposed algorithm substitutes a faster histogram processing for the strict sort processing in Ref. [4], and the details are explained in Section 2. The simulated and experimental results are presented and discussed in Sections 3 and 4, which indicate that the quasi-real-time performance is achieved and the proposed algorithm is proved to be superior to Herráez's algorithm [4] in terms of precision and execution time. Finally the paper is summarized in Section 5.

2. Algorithm

In quality guided path algorithm there are two main issues: the quality quantization of the pixel and the design of the unwrapping path. The proposed algorithm solves the first issue by using reliability function which has a fixed value range. Depending on unwrapping the high-quality pixels in preference to the low-quality pixels, the proposed algorithm determines the unwrapping path by histogram processing technique but not by the strict sorting of the reliability values. This approach not only is more efficient computationally but also reacts satisfactorily to noisy areas and discontinuities in practice. The following section explains the principle of this algorithm.

3. Reliability function

The criterion to determine the reliability value of a pixel is critical. This criterion is usually based on first or second differences between a pixel and its neighbors. In terms of accuracy, the use of first differences has some disadvantages. If the object is tilted severely, a high carrier value is present and becomes the major modulation component, even when the object is flat. This will produce an inappropriate measurement for the reliability values of pixels. Second differences can provide a better quantization for this situation. Moreover, second differences would provide a measurement for the degree of concavity/convexity for the zone with high curvature. So a better detection of possible inconsistencies in the wrapped phase map is provided by using second differences.

The proposed algorithm is based on second differences. The calculation of reliability values for pixels in an image can be explained with the aid of Fig. 1. To calculate the reliability value for a pixel in an image, a set of 8 pixels, called the 8-neighbors of (i, j), is required. The pixels $(i, j-1)$, $(i, j+1)$, $(i-, j)$, and $(i+1, j)$ that are neighbors to the pixel (i, j) are called orthogonal neighboring pixels, whereas $(i-1, j-1)$, $(i+1, j-1)$, $(i-1, j+1)$, and $(i+1, j+1)$ pixels are called diagonal neighboring pixels. The reliability value R of an (i, j) pixel can be calculated by the values of second differences D in the equation:

$$R = \frac{1}{D}$$

$(i-1, j-1)$	$(i, j-1)$	$(i+1, j-1)$
$(i-1, j)$	(i, j)	$(i+1, j)$
$(i-1, j+1)$	$(i, j+1)$	$(i+1, j+1)$

Fig. 1. Calculation of the reliability values for pixels in an image.

where

$$D(i,j) = [H^2(i,j) + V^2(i,j) + D_1^2(i,j) + D_2^2(i,j)]^{1/2} \quad (1)$$

or for the purpose of reducing the computational cost

$$D(i,j) = H^2(i,j) + V^2(i,j) + D_1^2(i,j) + D_2^2(i,j) \quad (2)$$

where

$$H(i,j) = \mathrm{wrap}[\varphi(i-1,j) - \varphi(i,j)] - \mathrm{wrap}[\varphi(i,j) - \varphi(i+1,j)]$$

$$V(i,j) = \mathrm{wrap}[\varphi(i,j-1) - \varphi(i,j)] - \mathrm{wrap}[\varphi(i,j) - \varphi(i,j+1)]$$

$$D_1(i,j) = \mathrm{wrap}[\varphi(i-1,j-1) - \varphi(i,j)] - \mathrm{wrap}[\varphi(i,j) - \varphi(i+1,j+1)]$$

$$D_2(i,j) = \mathrm{wrap}[\varphi(i-1,j+1) - \varphi(i,j)] - \mathrm{wrap}[\varphi(i,j) - \varphi(i+1,j-1)]$$

where wrap$((\cdot))$ is a simple modulo 2π operation to remove any 2π steps between two consecutive pixels.

The reliability values R can be calculated for all pixels in an image except the borders because some of the pixels in 8-neighbors fall outside the image if (i, j) is on the border of the image. The reliability values R of the pixels at the borders of the image are set to maximum 4π for (1) or $16\pi^2$ for (2) conservatively, to be resolved last as will be explained in the following subsection.

However, there are other definitions of the reliability value R. While reliability value R is the reciprocal of second differences value D, the definition of D could be different. There are three other definitions of D.

$$D(i,j) = \max(H*H, V*V, D_1*D_1, D_2*D_2)$$

$$D(i,j) = \max(|H|, |V|, |D_1|, |D_2|)$$

$$D(i,j) = |H| + |V| + |D_1| + |D_2|$$

Many experiments show that these different definitions of D share an almost same result with the original one but are slower than the original one when executed in their respective programs.

Consequently, pixels are more reliable as their reliability values R are higher which means their second differences values D are lower.

Because each unwrapping operation is carried out between two adjacent pixels, a basic structure called edge is defined reasonably, which is an intersection of two pixels that are connected horizontally or vertically. Every two orthogonal neighboring pixels can construct an edge. The reliability value of an edge is defined as the summation of the reliability values of the two pixels that the edge connects.

It can be deduced that the defined reliability function for pixels has a fixed value range $[0,4\pi]$ for (1) or $[0,16\pi^2]$ for (2), and the range of second differences values of edges is $[0,8\pi]$ or $[0,32\pi^2]$ respectively. We use (2) in the following subsection. As reliability values are more likely to be a fraction, for the purpose of reducing computational cost and convenience the proposed algorithm unwraps the phase maps depending on second differences values instead of reliability values.

4. Unwrapping procedure

Histogram processing is a popular tool for real-time image processing and can be used effectively for estimating the distribution of second differences values. The main advantage of using histogram processing is that this procedure needs only a "linear"