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Self-focusing of super-Gaussian laser beam inmagnetized plasma under relativistic and ponderomotive regime

Tarsem Singh Gill, Ravinder Kaur, Ranju Mahajan*

Department of Physics, Guru Nanak Dev University, Amritsar 143005, India

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ABSTRACT

In this paper, we have investigated the propagation characteristics of super-Gaussian laser beam in magnetized plasma. Self-focusing of short pulse laser propagating along the direction of ambient magnetic field in plasma is studied. The nonlinearity arises through the combined effect of relativistic mass variation and ponderomotive force induced electron cavitation. An appropriate nonlinear Schrödinger equation has been solved analytically using variational approach. Self-phase modulation is also studied under variety of parameters. Further, the effect of magnetic field on self-focusing of the beam have been explored.

1. Introduction

The interaction of high intensity laser radiation with plasma is an extensively studied physical phenomena. Propagation of short intense laser pulses through plasma is related to wide range of applications such as plasma based accelerator [1–4], optical harmonic generation [5,6], X-ray lasers [7,8], new radiation sources [9–12] and inertial confinement fusion with fast ignitor scheme [13,14]. This has given a boost to study propagation characteristics of high power lasers in plasma medium. Thus for the success of above mentioned applications it is only desirable that laser beam propagate over sufficient number of Rayleigh lengths. But in vacuum, laser propagation is limited by the diffraction process, the characteristic distance of which is Rayleigh length $Z_d = \frac{\pi r_0^2}{\lambda}$, where λ is laser wavelength and r_0 is spot size. However, such situation is prone to large number of instabilities and other undesirable effects [15]. Self-focusing and filamentation are genuinely nonlinear basic physical mechanisms and plays crucial role in propagation of lasers in underdense plasma [16–18]. Self-focusing is a nonlinear optical process induced by a change in refractive index of material expose to intense laser radiation. A medium whose refractive index increases with electricfield intensity act as focusing lens for a laser beam characterized by an initial transverse intensity gradient. Advances in laser technology have enabled observation of self-focusing in the interaction of intense laser pulses with plasma. Basic nonlinear physical mechanisms which play crucial role in self-focusing phenomenon are collisional, ponderomotive, relativistic, heating type as reported in the research work [19,20]. For example, thermal self-focusing is due to collisional heating of a plasma exposed to electromagnetic radiation and rise in temperature induced hydrodynamic expansion which leads to increase in index of refraction and further heating. Ponderomotive nonlinearity, resulting from intensity gradient of laser beam, is operational on the time scale of $\frac{a_0}{v_s}$, where a_0 is the dimension of the beam and v_s is the ion acoustic speed. As very high power laser beams are used in experiments in these days, the quiver motion also reduces the local plasma frequency, resulting in relativistic self-focusing [21–23]. Experimental as well as theoretical observations of relativistic self-focusing and ponderomotive self-channeling have been reported in a number of investigations [22–29]. The dynamics of ponderomotive channeling in underdense plasma has recently been studied experimentally [30]. Relativistic laser plasma interaction physics has also been focus of attention as many nonlinear processes playing key roles in the generation of new ion sources as reported recently [31,32]. The presence of ambient magnetic field in the plasma was seen to influence the self-focusing very significantly. Stenzel [33,34] observed self-focusing and filamentation of whistlers in a large magnetized plasma device over wide range of parameters.

With the advent of intense short pulse laser, a new regime of nonlinear laser plasma interaction became important where ions during the passage of the pulse could be treated as immobile. All the experiments on laser driven acceleration via beat wave and wake field excitation of plasma wave as well as direct laser acceleration employ laser pulse much shorter than the ion plasma period and ions may be treated as immobile. In such situation the space

* Corresponding author. Tel.: +91 8146292032.
E-mail addresses: gillsema@yahoo.co.in (T.S. Gill), physics_ravinder@yahoo.co.in (R. Kaur), mahajan_ranju@yahoo.co.in (R. Mahajan).

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charge field, due to imbalance of electron and ion space plays an important role in the dynamics of accelerating electrons. Shukla and Stenflo [35] also carried out investigations of electromagnetic waves in magnetized plasma. Much work has been done on relativistic self-focusing [36–38]of short pulse laser in plasma where nonlinearity arises due to relativistic mass variation of electrons and ponderomotive force induced electron density depletion, leaving ion static. Mostly used model to study self-focusing is based on WKB and paraxial ray approximation (PRA) through a nonlinear parabolic wave equation [17,19]. Liu and Tripathi used PRA and WKB approximation to study the competing physical process of self-focusing and diffraction. The main drawback of the PRA is that it overemphasizes the importance of field close to beam axis and lacks global pulse dynamics. Another global approach is variational approach, though crude to describe the singularity formation and collapse dynamics is fairly genuine in nature to study propagation and also it correctly predicts the phase.

Most of the theoretical investigations of self-focusing of laser beam in nonlinear medium including plasmas have been carried out for simple Gaussian beam [39–41] and cylindrically symmetric Gaussian beam [42–45]. Only a few investigations have been reported on self-focusing of super Gaussian [46–49], self trapping of degenerate modes of laser beam [44], self-trapping of Bessel beam [50], elliptic Gaussian beam [51–54], hollow elliptic Gaussian beam [55], Hermite-cosh-Gaussian beam [56] and cosh Gaussian spiral field [57–59]. Focusing of dark hollow Gaussian electromagnetic beams in plasma has been reported [60]. Ring formation in electromagnetic beams in relativistic magnetoplasma is given by [61]. Most previous investigations [62,63] based on variational approach have used Gaussian or sech expressions as trial functions for the radial shape of fundamental nonlinear mode. The Gaussian and especially the sech solutions indeed exhibit good agreement with the numerically obtained results for the cases considered. These trial functions do not however, provide any information about how radial shape of the fundamental mode depends on pulse amplitude, pulse width, distance of propagation, degree of nonlinearity etc. One way of allowing for a more flexible shape to choose super-Gaussian trial functions. In present investigation, authors have studied evolution of super-Gaussian beam in a plasma when relativistic and ponderomotive nonlinearities are considered.

The paper is organized as follows. In Section 2 we have given brief description of dielectric constant and derived the equation for the beam width parameter a_n using variational approach in magnetized plasma including the effect of short time scale ponderomotive force and relativistic nonlinearity. In Section 3, we presented a detailed discussion of numerical work carried out for the relevant parameters. LastSection 4 is devoted to the conclusions of the present investigation.

2. Basic formulation

A laser with a Gaussian profile along its wavefront exerts a ponderomotive force on electrons $\vec{F_p} = e\nabla\phi_p$, where

$$\phi_p = \frac{-m_0c^2}{e\left[\left(1+\frac{e^2|A^2|}{m_0^2c^2\left(\omega-\frac{\omega_c}{\gamma}\right)^2}\right)^{\frac{1}{2}}-1\right]} \tag{1}$$

is produced creating a space charge field $\vec{E_s} = -\nabla\phi_s$. In the quasi-steady state $\phi_s = -\phi_p$, employing Poisson's equation $-\nabla^2\phi_s = 4\pi e(n-n_i)$, where $n_i = n_0$ is the ion density and n is the modified electron density, we may write

$$n = n_0 + \frac{1}{4\pi e}\nabla^2\phi_p \tag{2}$$

$$\frac{n}{n_0} = \left[1-\frac{c^2}{\omega_p^2}\nabla^2\left(1+\frac{e^2|A^2|}{m_0^2c^2\left(\omega-\frac{\omega_c}{\gamma}\right)^2}\right)^{\frac{1}{2}}\right] \tag{3}$$

Here, ω_p is the unmodified plasma frequency as defined above. The non linear permittivity [64] may be written as:

$$\epsilon_+(r,z) = 1-\frac{\omega_p^2}{\omega\left(\omega-\frac{\omega_c}{\gamma}\right)\gamma}\frac{n}{n_0} \tag{4}$$

$$\epsilon_+(r,z) = 1-\frac{\omega_p^2}{\omega\left(\omega-\frac{\omega_c}{\gamma}\right)\gamma}\left[1-\frac{c^2}{\omega_p^2}\nabla^2\left(1+\frac{e^2|A^2|}{m_0^2c^2\left(\omega-\frac{\omega_c}{\gamma}\right)^2}\right)^{\frac{1}{2}}\right] \tag{5}$$

$$\epsilon_+(r,z) = 1-\frac{\omega_p^2}{\omega\left(\omega-\frac{\omega_c}{\gamma}\right)\gamma}+\frac{c^2}{\omega^2\gamma\left(1-\frac{\omega_c}{\gamma\omega}\right)}\nabla^2 \times\left(1+\frac{e^2|A^2|}{2m_0^2c^2\omega^2}\left(1+5\frac{\omega_c}{\omega}\right)-2\frac{\omega_c}{\omega}\frac{e^4|A^4|}{m_0^4c^4\omega^4}\right) \tag{6}$$

As relativistic factor is given by:

$$\gamma = \left[1+\frac{e^2|A^2|}{m_0^2c^2\left(\omega-\frac{\omega_c}{\gamma}\right)^2}\right]^{\frac{1}{2}} \tag{7}$$

$$\gamma = \left[1+\frac{a^2}{\left(1-\frac{\omega_c}{\gamma\omega}\right)^2}\right]^{\frac{1}{2}} \tag{8}$$

where

$$a^2 = \frac{e^2|A^2|}{m_0^2c^2\omega^2} \tag{9}$$

For $\frac{\omega_c}{\gamma\omega} < 1$, the equation can be solved iteratively. First we choose $\omega_c = 0$, then $\gamma = (1+a^2)^{\frac{1}{2}}$; further using this value of γ in the right hand side of Eq. (8), we obtain

$$\gamma = \left[1+a^2\left(1-\frac{\omega_c}{\omega(1+a^2)^{\frac{1}{2}}}\right)^{-2}\right]^{\frac{1}{2}} \tag{10}$$

$$\gamma = \left[1+a^2+2a^2\frac{\omega_c}{\omega}\left(\frac{1}{(1+a^2)^{\frac{1}{2}}}\right)+3a^2\frac{\omega_c^2}{\omega^2}\left(\frac{1}{(1+a^2)}\right)\right]^{\frac{1}{2}} \tag{11}$$

After solving we get:

$$\gamma = \left(1+\frac{e^2|A^2|}{2m_0^2c^2\omega^2}\left(1+5\frac{\omega_c}{\omega}\right)-2\frac{\omega_c}{\omega}\frac{e^4|A^4|}{m_0^4c^4\omega^4}\right) \tag{12}$$

The wave equation governing the propagation of electromagnetic wave in extraordinary mode:

$$\nabla_\perp^2A_+ - 2\iota k\frac{\partial A_+}{\partial z}+\frac{\omega^2}{c^2}\left[1-\frac{\omega_p^2}{\omega\left(\omega-\frac{\omega_c}{\gamma}\right)\gamma}+\frac{c^2}{\omega^2\gamma\left(1-\frac{\omega_c}{\gamma\omega}\right)}\nabla_\perp^2 \times\left(1+\frac{e^2|A_+^2|}{2m_0^2c^2\omega^2}\left(1+5\frac{\omega_c}{\omega}\right)-2\frac{\omega_c}{\omega}\frac{e^4|A_+^4|}{m_0^4c^4\omega^4}\right)\right] = 0 \tag{13}$$

The exact solution of Eq. (13) is not available and we therefore seek either numerical or analytical approximate method. Although