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Spreading of apertured partially coherent beams in turbulent media

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ABSTRACT

The closed-form expression for the mean-squared width of apertured partially coherent beams propagating through turbulent media is derived by using the integral transform technique. The influence of turbulence on the spreading of apertured partially coherent beams is studied quantitatively by examining the relative mean-squared width, which is defined as the ratio of the mean-squared width of an apertured partially coherent beam in turbulence to the mean-squared width of the same beam in free space. On the other hand, the range of turbulence-independent propagation, also a reasonable measure of the resistance of a beam to turbulence, is obtained by examining the mean-squared width. It is shown that the spreading of apertured partially coherent beams is less affected by turbulence with smaller truncation parameter δ and coherence parameter α than with larger δ and α. In addition, the influence of turbulence on the spreading of apertured partially coherent beams increases first and then decreases due to increasing waist width w_0. The results obtained are explained physically.

1. Introduction

It is of interest in studying propagation properties of laser beams in turbulence for many practical applications such as tracking, remote sensing and atmospheric optical communication, etc [1]. One of the important properties of laser beams propagating through turbulent media is their spreading, which has been carried out widely, such as the spreading of Gaussian Schell-model (GSM) beams, partially coherent Hermite–Gaussian (HG) beams, and it was demonstrated that partially coherent beams are less affected by turbulence than fully coherent ones [2–6]. In 2003 Shirai et al. [7] pointed out that in atmospheric turbulence the relative spreading of higher-order modes is smaller than that of lower-order modes. In 2009 we demonstrated that annular beams with larger obscure ratio ε, larger order M, larger wave length λ, and smaller outer radius w_0 are less sensitive to the turbulence than those with smaller ε, M, λ and larger w_0 [8].

On the other hand, it is well-known that the beam emitted from a laser system is more or less apertured in practice. As yet, Refs. [9–13] have dealt with the propagation property of apertured laser beams through atmospheric turbulence. Recently, we studied the directionality of apertured GSM beams propagating through atmospheric turbulence [14]. However, the spreading of apertured partially coherent beams propagating through atmospheric turbulence has not been examined until now.

The goal of this work is to investigate the spreading of apertured partially coherent beams propagating through turbulent media. The closed-form expression for the mean-squared width of apertured partially coherent beams is derived. The influence of turbulence on the spreading of apertured partially coherent beams is studied quantitatively by examining two relevant parameters, i.e., the relative mean-squared width $\left(w(z)_{\text{turb}}/w(z)_{\text{free}}\right)$ and the turbulence distance z_T. Some interesting results are obtained and interpreted physically.

2. Relative mean-squared width

We suppose a Gaussian Schell-model (GSM) beam is incident upon a slit with full width $2d$ oriented along x axis at the source plane $z = 0$. The rectangular function of the form

$$T(x) = \begin{cases} 1 & |x| \le d \\ 0 & |x| > d \end{cases} \tag{1}$$

can be used to describe the window function of the slit, and $T(x)$ can be expanded into a finite sum of complex-valued Gaussian functions [15]

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$$T(x)=\sum_{i=1}^{M}F_i\ \exp\left(-\frac{G_i x^2}{d^2}\right) \tag{2}$$

where the coefficients F_i, G_i and the number M are evaluated by a computation fitting [15], and omitted here. It shows that the method of the finite complex Gaussian expansion of the aperture function is applicable to the Fraunhofer and Fresnel regions except for the extreme near field (<0.1 times the Fresnel distance) [15].

In the Cartesian coordinate system, the cross-spectral density function of a GSM beam at the source plane ($z=0$) can be expressed as [16]

$$W^{(0)}(x'_1,x'_2,z=0)=\ \exp\left(-\frac{x'^2_1+x'^2_2}{w_0^2}\right)\ \exp\left[-\frac{\left(x'_1-x'_2\right)^2}{2\sigma_0^2}\right] \tag{3}$$

where w_0 and σ_0 are the waist width and spatial correlation length at the source plane $z=0$, respectively.

Based on the extended Huygens–Fresnel principle, the average intensity of an apertured GSM beam propagating through turbulent media reads as

$$\begin{aligned}\left\langle I(x,z)\right\rangle &= \frac{k}{2\pi z}\int_{-d}^{d}\int_{-d}^{d}dx'_1 dx'_2 W^{(0)}\quad (x'_1,\quad x'_2,\quad z=0)\\ &\times\exp\left\{\left(\frac{ik}{2z}\right)\left[(x'^2_1-x'^2_2)-2(x'_1-x'_2)x\right]\right\}\left\langle\exp\left[\psi^*(x'_1,x,z)+\psi(x'_2,x,z)\right]\right\rangle_m\end{aligned} \tag{4}$$

where $k=\left(2\pi/\lambda\right)$ (λ is the wavelength), $\psi(x',x,z)$ denotes the random part of the complex phase of a spherical wave that propagates from the source point to the receiver point, $\langle\rangle_m$ denotes average over the ensemble of the turbulent medium, and [17]

$$\left\langle\exp\left[\psi^*(x'_1,x,z)+\psi(x'_2,x,z)\right]\right\rangle_m\cong\exp\left[-\frac{(x'_1-x'_2)^2}{\rho_0^2}\right] \tag{5}$$

with

$$\rho_0=(0.545C_n^2k^2z)^{-3/5} \tag{6}$$

where ρ_0 denotes the spatial coherence radius of a spherical wave propagating through turbulence, and C_n^2 specifies the refraction index structure constant. Rytov's quadratic approximation of the phase structure function, a suitable approximation in practice, is used in Eq. (5).

Substituting Eqs. (2) and (5) into Eq. (4), Eq. (4) can be written as

$$\begin{aligned}\left\langle I(x,z)\right\rangle &= \frac{k}{2\pi z}\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dx'_1dx'_2W^{(0)}\quad(x'_1,\quad x'_2,\quad z=0)\\ &\times\ \exp\left(-\frac{G_ix'^2_1}{d^2}\right)\exp\left(-\frac{G_j^*x'^2_2}{d^2}\right)\\ &\exp\left\{\left(\frac{ik}{2z}\right)\left[(x'^2_1-x'^2_2)-2(x'_1-x'_2)x\right]\right\}\\ &\times\ \exp\left[-\frac{(x'_1-x'_2)^2}{\rho_0^2}\right]\end{aligned} \tag{7}$$

Considering Eq. (3) and recalling the integral formula,

$$\int_{-\infty}^{\infty}\exp\left(-A^2x^2+Bx\right)dx=\frac{\sqrt{\pi}}{A}\exp\left(\frac{B^2}{4A^2}\right) \tag{8}$$

After tedious integral calculations, the final result can be arranged as

$$\left\langle I(x,z)\right\rangle=\frac{k}{2z}\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{F_iF_j^*}{\beta}\exp\left[-\frac{k^2}{4\beta^2z^2}\left(\frac{G_i+G_j^*}{w_0^2\delta^2}+\frac{2}{w_0^2}\right)x^2\right] \tag{9}$$

where

$$\begin{aligned}\beta=&\left[\frac{1}{w_0^4}+\frac{k^2}{4z^2}+\frac{G_iG_j^*}{w_0^4\delta^4}+\frac{ik}{2z}\left(\frac{G_i-G_j^*}{w_0^2\delta^2}\right)+\frac{G_i+G_j^*}{w_0^2\delta^2}\left(\frac{1}{w_0^2}+\frac{1}{2w_0^2\alpha^2}+\frac{1}{\rho_0^2}\right)\right.\\ &\left.+\frac{2}{w_0^2}\left(\frac{1}{2w_0^2\alpha^2}+\frac{1}{\rho_0^2}\right)\right]^{1/2}\end{aligned} \tag{10}$$

here $\delta=\left(d/w_0\right)$ and $\alpha=\left(\sigma_0/w_0\right)$ are called the beam truncation parameter and coherence parameter [16], respectively. From Eq. (9), it can be seen that the average intensity distribution of apertured GSM beams depends on C_n^2, δ, α, w_0, z and λ.

If $\delta\to\infty$, Eq. (9) reduces to

$$\left\langle I(x,z)\right\rangle|unapertured=\frac{k}{2z\gamma}\exp\left[-\frac{k^2}{2w_0^2z^2\gamma^2}x^2\right] \tag{11}$$

where

$$\gamma=\left[\frac{1}{w_0^4}+\frac{2}{w_0^2}\left(\frac{1}{2w_0^2\alpha^2}+\frac{1}{\rho_0^2}\right)+\frac{k^2}{4z^2}\right] \tag{12}$$

Eq. (11) is the average intensity of unapertured GSM beams propagating through turbulent media.

The mean-squared beam width is defined as [2]

$$w^2(z)=\frac{2\int_{-\infty}^{\infty}x^2I(x,z)dx}{\int_{-\infty}^{\infty}I(x,z)dx} \tag{13}$$

On substituting Eq. (9) into Eq. (13), and making use of the integral transform technique, the mean-squared beam width of apertured partially coherent beams propagating through turbulent media described by Eq. (13) turns out to be

$$w^2(z)=P+Qz+Rz^2+Fz^{16/5} \tag{14}$$

where

$$P=\frac{\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*\left(\left(\left(G_i+G_j^*\right)/\delta^2\right)+2\right)^{-3/2}\times w_0^2}{\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*\left(\left(\left(G_i+G_j^*\right)/\delta^2\right)+2\right)^{-1/2}} \tag{15}$$

$$Q=\frac{2i}{k}\times\frac{\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*\left(\left(\left(G_i+G_j^*\right)/\delta^2\right)+2\right)^{-3/2}\left(\left(G_i-G_j^*\right)/\delta^2\right)}{\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*\left(\left(\left(G_i+G_j^*\right)/\delta^2\right)+2\right)^{-1/2}} \tag{16}$$

$$R=\frac{4}{k^2w_0^2}\times\frac{\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*\left((G_i+G_j^*)/\delta^2+2\right)^{-3/2}\times\left[G_iG_j^*/\delta^4+\left(1+1/2\alpha^2\right)\left((G_i+G_j^*)/\delta^2\right)+1+\left(1/\alpha^2\right)\right]}{\sum_{i=1}^{M}\sum_{j=1}^{M}F_iF_j^*\left(\left((G_i+G_j^*)/\delta^2\right)+2\right)^{-1/2}} \tag{17}$$

$$F=4\times\left(0.545C_n^2\right)^{6/5}k^{2/5} \tag{18}$$