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Estimating ultimate bound and finding topological horseshoe for a new chaotic system

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ABSTRACT

This paper presents a new three-dimensional autonomous chaotic system with only one positive term. Basic dynamical properties of the new attractor are demonstrated in terms of phase portraits, equilibria, Lyapunov exponents, Poincare mapping, bifurcation diagram. Furthermore, we derive a three-dimensional spheriform ultimate bound and positively invariant set for all the positive values of its parameters *a*, *b*, *c*. At last, the horseshoe chaos in this system is investigated based on the topological theory.

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1. Introduction

Since the discovery of the Lorenz chaotic system in 1963 [1], chaos has been studied extensively. As the first chaotic model, Lorenz had declared that chaotic systems are dynamic systems described by nonlinear differential equations and they are strongly sensitive to initial conditions, which means that, even if the system mathematical description is deterministic, its behavior is still unpredictable. From then on, many-like chaotic systems such as Rössler systems [2], Chen system [3]. Lü system and Liu systems [4,5] were reported and analyzed. And in 2002, a unified chaotic system was created that bridges Chen system to Lorenz system through Lü system chaotic attractor [6]. It is notable that the family of Lorenz systems has two quadratic terms on the right-hand side of the governing equations. More recently, Qi et al. [7] introduce a new three-dimensional smooth autonomous chaotic system with three quadratic terms. When proper parameters are chosen, a single four-wing attractor and two coexisting single-wing attractors with different initial can be generated.

Due to great potential in chemical reactions, cryptology, electrical engineering, information processing and so on, it is important to generate new chaotic systems and analyze their dynamical behaviors and dynamical properties.

As we know, though a chaotic system is bounded, it is not an easy work to estimate and examine its bound. Therefore, an interesting fundamental question is how to estimate the bound of strange attractor. However, the ultimate bounds of many other chaotic system remain to be solved. The ultimate bound of Chen system is investigated in [8], but the parameter values considered does not cover the most interesting case of the Chen's chaotic attractor. And the ultimate bound of the Lü system has not been studied yet. The bound estimation of a chaotic system is also a challenging task for most known chaotic systems, even for the classical Lorenz system. Generally speaking, there are mainly four methods to estimate the bounds of chaotic systems in current literature, which is the hyper-plane oriented method [9], the iteration theorem and the first order extremum theorem [10], Lyapunov stability theory combined with the comparison principle method [11], and the optimization method [12,13]. Among which the latter two methods are proved to be effective and simpler.

Topological horseshoe with symbolic dynamics is a powerful tool in rigorous studies of chaos in dynamical systems. Up to now, remarkable theoretical progress has been made in seeking sufficient conditions for the existence of horseshoes. Kennedy introduced an important chaos lemma which proposed a topological horseshoe theory in continuous map [14,15]. Yang obtained another concernful criteria to find the topological horseshoe in non-continuous map [16,17], which have been applied successfully to some practical dynamical systems to present computer-assisted verification of chaos [18–23]. Recently, Li introduced a new method for finding horseshoes in chaotic systems by using several simple results on topological horseshoes [24]. However, it is still a challenge for researchers to seek a topological horseshoe in practical chaotic systems.

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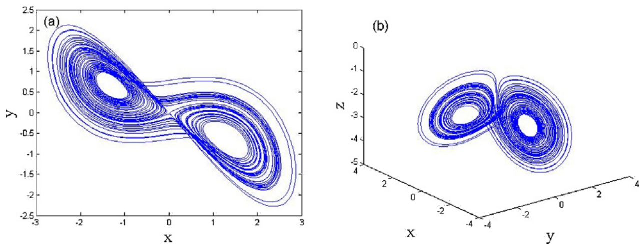

Fig. 1. (a) x–y phase plane (b) three-dimensional view.

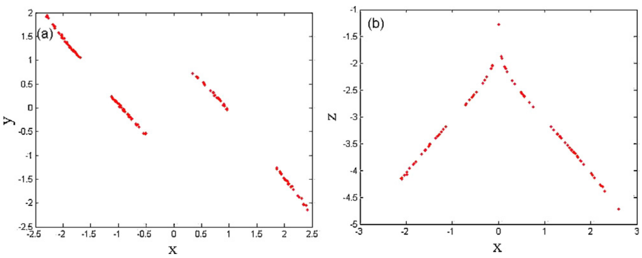

Fig. 2. (a) The Poincare mapping of x–y plane (b) the Poincare mapping of x–z plane.

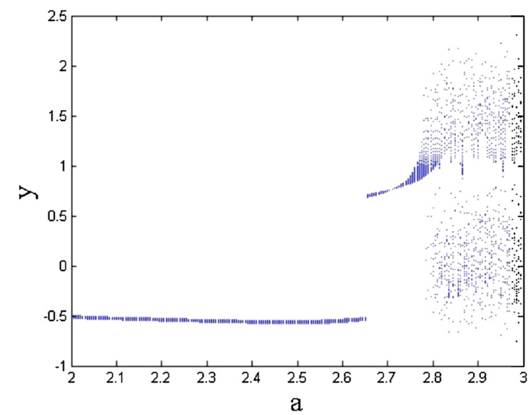

Fig. 3. The bifurcation diagram with respect to a.

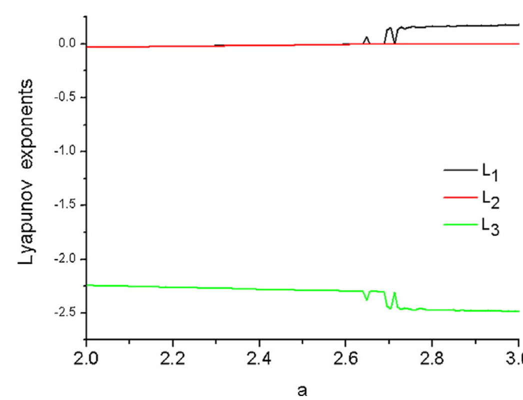

Fig. 4. The Lyapunov exponents spectrum with respect to a.

2. System description and dynamic properties

In this paper, a new 3D chaotic system is written as follows:

$$\begin{aligned} \dot{x} &= -x - 2y, \\ \dot{y} &= -xz - by - ax, \\ \dot{z} &= xy - cz, \end{aligned} \tag{1}$$

where x, y, z denote the state variables, a, b, c are the positive real numbers. Clearly, the new system with only one positive term xy is different form the Lorenz system.

2.1. Equilibria and stability

It is known that the number of system equilibrium and their stabilities are very important for the emergence of chaos. In the following, we consider the equilibrium of system (1). For this purpose, let

$$-x - 2y = 0, \quad -xz - by - ax = 0, \quad xy - cz = 0. \tag{2}$$

Then, if $2a \geq b,\ c > 0$, we get three equilibria of system (1):

$$P_0(0,0,0),\ P_1\left(\sqrt{2(2a-b)c}, -\sqrt{\frac{(2a-b)c}{2}}, \frac{b-2a}{2}\right),$$

$$P_2\left(-\sqrt{2(2a-b)c}, \sqrt{\frac{(2a-b)c}{2}}, \frac{b-2a}{2}\right).$$

Linearizing system (1) at any equilibrium (x_{10}, y_{20}, z_{30}), it yields the corresponding Jacobian matrix

$$J = \begin{bmatrix} -1 & -2 & 0 \\ -z_{30} - a & -b & -x_{10} \\ y_{20} & x_{10} & -c \end{bmatrix}.$$

The characteristic equation is get as below:

$$f(\lambda) = \lambda^3 + C_2\lambda^2 + C_1\lambda + C_0, \tag{3}$$

where

$$\begin{aligned} C_0 &= bc - 2x_{10}y_{20} - 2ac - 2cz_{30} + x_{10}^2, \\ C_1 &= bc + x_{10}^2 + b + c - 2z_{30} - 2a, \\ C_2 &= b + c + 1. \end{aligned} \tag{4}$$

According to the Routh–Hurwitz criterion, only if $C_2 > 0$, $C_1 > 0$, $C_0 > 0$ and $C_2C_1 - C_0 > 0$, the real parts of all the roots are negative. Thus, there are three unstable equilibria in system (1) when $a = 3$, $b = 0.3$, and $c = 0.3$.

2.2. Chaotic phase portraits, Poincare mapping

When $a = 3$, $b = 0.3$, $c = 0.3$, system (1) is chaotic with Lyapunov exponents $L_1 = 0.1792$, $L_2 = 0$, $L_3 = -2.487$. The phase portraits are depicted in Fig. 1. Furthermore, Poincare sections also show this system is chaotic. The numerical result is displayed in Fig. 2, where two Poincare sections on the x–y and x–z planes are depicted.

2.3. Lyapunov exponent spectrum and bifurcation diagram

The numerical features of the new chaotic attraction can be further illustrated by its Lyapunov exponent spectrum and its bifurcation diagram, as shown in Figs. 3 and 4, respectively.

3. Spheriform localization with precise bounds

Theorem 1. *When $a > 0$, $b > 0$, $c > 0$, the following three-dimensional spheriform set $\Omega = \{(x, y, z) | x^2 + y^2 + (z + a + 2)^2 \leq R^2\}$ is the ultimate bound and positively invariant set of system (1)*

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