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A novel ray refraction matrix for curved interface

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ABSTRACT

A problem in fundamental 2×2 ray matrix for refraction at curved interface proposed by A. E. Siegman was found out and a novel one was derived in this paper. An experiment is introduced in detail to verify the reasonability of the novel ray matrix. Using the novel 2×2 ray matrix, augmented 5×5 ray matrix of refraction at misaligned curved interface between media of different refractive indices was deduced. With the refraction matrix, it is easy to characterize the effect of an astigmatic thick lens. The augmented ray matrix approach was applied to model and estimate the performance of an optical alignment system. Utilizing these matrices, one can readily design and evaluate optical systems, where contain astigmatic elements such as tilted spherical or cylindrical lenses, mirrors and so on. These results are also useful for cavity design, alignment, ray tracing and beam position control in 3D optical systems.

1. Introduction

Ray matrix is useful for cavity design, alignment, ray tracing and beam position control in 3D optical systems comprising of spherical refracting surfaces [1–7]. A 2×2 ray matrix is used to represent each optical operation on the ray of light. In 1975, Gerrard and Burch presented a 3×3 matrix formalism that was expanded to handle image translation and slope addition [3]. 4×4 matrix formalism was presented by Siegman that allowed orthogonal axes to be modeled so that things such as simple astigmatism and image rotation can be modeled [1]. Harris [4] presented a 5×5 ray matrix formalism that enables modeling of effects such as slope addition, image translation, and image rotation. A novel coordinate system of ray reflection was proposed to modify the inconsistency between traditional coordinate system and ABCD matrix for reflection [5]. Based on this novel coordinate system, generalized augmented matrix for misaligned spherical mirror reflection was derived [6]. However, to the best of our knowledge, augmented 5×5 ray matrix for refraction at misaligned curved interface has not been derived before and it should be based on the fundamental 2×2 ray matrix for refraction at misaligned curved interface. Unfortunately, a similar problem exists in the fundamental 2×2 ray matrix for refraction at curved interface which was proposed by Siegman [1]. The problem is that traditional ray matrix is inconsistent with its corresponding coordinate system and incorrect position will be obtained using the fundamental 2×2 ray matrix for refraction at curved interface, which was proposed by A. E. Siegman. A novel 2×2 ray matrix for refraction at curved interface has been obtained and proved by experiment. Moreover, augmented 5×5 ray matrix for refraction at curved interface with all possible perturbation sources was derived with perturbation method through utilizing the correct 2×2 ray refraction matrix for the first time. As a typical 3D optical system, the performance of an optical alignment system was estimated with the augmented 5×5 ray matrix approach.

2. Ray matrix for refraction at curved surface

The reflection and refraction of a Gaussian beam at a curved interface between two media have been analyzed by Siegman [8], but the results in Ref. [8] was unsuitably transformed into the 2×2 ray matrix for refraction at curved surface [1]. The fundamental 2×2 ray matrix for refraction at curved interface in Ref. [1] (arbitrary incidence, in the plane of incidence) has the form of

$$M = \begin{bmatrix} \cos\,\theta_2/\cos\theta_1 & 0 \\ \Delta n_e/R & \cos\,\theta_1/\cos\theta_2 \end{bmatrix}, \tag{1}$$

where θ_1 is the angle between z_i axis and symmetric axis OO', θ_2 is the angle between z_t axis and symmetric axis OO' in Fig. 1(a)

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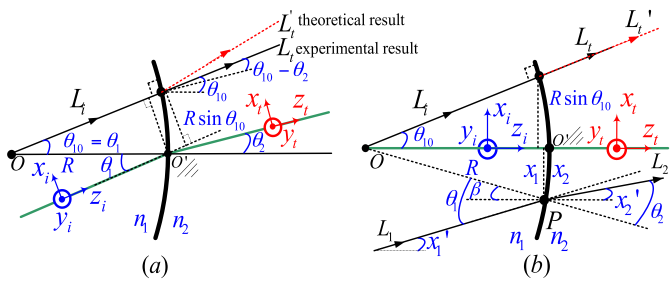

Fig. 1. A simple optical experiment of refraction at curved surface. O: spherical center of curved interface; O': the intersection of the curved surface and its symmetric axis; R is radius of the curved surface; n_1, n_2: refractive indexes of two media; θ_{10}: angle between L_i and line OO'; (a) (x_i, y_i, z_i) and (x_t, y_t, z_t): coordinate systems of the incident and refracted rays proposed by Siegman in Ref. [1]; the $z_j(j=i, t)$ axes point along the reference axis (in green solid line); the $x_j(j=i, t)$ and $z_j(j=i, t)$ form the incident plane; θ_1: angle between z_i and line OO'; θ_2: angle between z_t and line OO'; L_i: a special incident ray emitted from spherical center O, L_i is parallel to the z_i axis; L_t: the experimental refracted ray; L_t' : the theoretical refracted ray obtained by utilizing the ray matrix proposed by Siegman in Ref. [1]; (b) (x_i, y_i, z_i) and (x_t, y_t, z_t): coordinate systems of the incident and refracted rays; The $z_j(j=i, t)$ axes point along the reference axis (in green) which is the symmetrical axis of curved surface; $x_i(i=1, 2)$: ray height from the reference axis $z_j(j=i, t)$; x_i' $(i=1, 2)$: tilted angle that $L_j(j=1, 2)$ make with reference axis $z_j(j=i, t)$; L_i: a special incident ray as an example for Eq. (6) proposed in this paper, L_t: the experimental refracted ray; L_t' : the theoretical refracted ray obtained by utilizing the ray matrix in Eq. (6); θ_1: incident angle on the interface from the medium of index n_1; θ_2: refracted angle on the interface in the medium of index n_2; P: the incident point of L_1; β: the angle between OP and z_i axis. (For interpretation of the references to color in figure legend, the reader is referred to the web version of the article.)

and $\Delta n_e=(n_2\cos\theta_2-n_1\cos\theta_1)/(\cos\theta_1\cos\theta_2)$. R is the radius of the curved surface. A ray vector was commonly specified as $[\,x_i \quad n_i\sin\theta_i\,]^T$ where x_i and θ_i represent the displacement and slope of the ith ray with respect to the z axis in corresponding coordinate system [3]. The vector $[\,x_i \quad n_i\sin\theta_i\,]^T$ is used to describe the ray position with respect to the corresponding coordinate frame. n_i is the refractive index of the media in which the ray is traveling. Now we will show why the matrix shown in Eq. (1) is incorrect for ray refraction and then derive the correct one by ray tracing method.

As shown in Fig. 1(a), a simple experiment was introduced to prove the matrix shown in Eq. (1) is not suitable for ray refraction at curved surface. The coordinate systems of the incident beam and the reflected beam are illustrated with blue solid arrow and red solid arrow respectively, The $z_j(j=i, t)$ axes point along the reference axis (in green solid line) which should be stationary and consistent with ray matrix (1). The special ray L_i emits from spherical center O and points along the z_i axis, so L_i is normal to the tangent plane of curved surface. In other words, L_i incidents normally on the interface. Thus its refracted ray L_t will propagate in the same direction of L_i. Relative to the reference axis z_i in Fig. 1(a), the position of incidence ray L_i can be specified as the ray vector

$$L_i=[\,x_i \quad n_1\sin\theta_i\,]^T=[\,R\sin\theta_{10} \quad n_1\sin(\theta_{10}-\theta_1)\,]^T. \tag{2}$$

Relative to the reference axis z_t in the coordinate system (x_t, y_t, z_t) in Fig. 1(a), the position of experimental refracted ray L_t can be expressed as the ray vector

$$L_t=[\,x_t \quad n_2\sin\theta_t\,]^T=[\,R\tan\theta_{10}\cos\theta_2 \quad n_2\sin(\theta_{10}-\theta_2)\,]^T. \tag{3}$$

However, the position of theoretical refracted ray L_t' is inconsistent with L_t. The ray refraction at curved interface with the augmented 2×2 matrix (1) has the form as

$$L_t' = ML_i = \begin{bmatrix} \cos\theta_2/\cos\theta_1 & 0 \\ \Delta n_e/R & \cos\theta_1/\cos\theta_2 \end{bmatrix} \begin{bmatrix} R\sin\theta_{10} \\ n_1\sin(\theta_{10}-\theta_1) \end{bmatrix} = \begin{bmatrix} R\tan\theta_{10}\cos\theta_2 \\ \dfrac{n_2\sin(\theta_{10}-\theta_2)}{\cos\theta_1\cos\theta_2} \end{bmatrix}, \tag{4}$$

where L_t' is the ray vector of theoretical refracted ray. According to Eq. (4), the theoretical refracted beam L_t' can be expressed as the ray vector $R\tan\theta_{10}\cos\theta_2 \quad n_2\sin(\theta_{10}-\theta_2)/(\cos\theta_1\cos\theta_2)$ in the coordinate system (x_t, y_t, z_t) in Fig. 1(a) and the position of L_t' is inconsistent with the experimental result $L_t=[\,R\tan\theta_{10}\cos\theta_2 \quad n_2\sin(\theta_{10}-\theta_2)]^T$, so it is proved to be incorrect.

As shown in Fig. 1(b), the coordinate systems of the incident beam and the reflected beam are illustrated with blue solid arrow and red solid arrow respectively. To make the derivation easier, the $z_j(j=i, t)$ axes point along the symmetric axis of the curved interface (in green solid line). A general paraxial incidence ray L_1 in the incidence plane (tangential plane) can be specified as the ray vector $[\,x_1 \quad n_1\sin x_1'\,]^T$ and its refracted ray L_2 can be expressed as $[\,x_2 \quad n_2\sin x_2'\,]^T$. One can easily find that

$$\begin{aligned} &\theta_1=\beta+x_1', \quad \theta_2=\beta+x_2', \\ &x_1=x_2, \quad \sin\beta=x_1/R. \end{aligned} \tag{5}$$

It is noteworthy that the definitions of θ_1 and θ_2 in Fig. 1(b) are different from the definitions in Fig. 1(a). As shown in Fig. 1(b), θ_1 is the incident angle on the interface from the medium of index n_1, θ_2 is the refracted angle on the interface in the medium of index n_2. β is the angle between OP and z_i axis. The refraction at the curved interface can be expressed as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_1 \\ n_1\sin x_1' \end{bmatrix} = \begin{bmatrix} x_2 \\ n_2\sin x_2' \end{bmatrix}. \tag{6}$$

After substituting Eq. (5) into Eq. (6), the results can be obtained:

$$A=1, B=0, D=1, C=\frac{(n_2\cos\theta_2-n_1\cos\theta_1)}{R}, \tag{7}$$

so the ray matrix for refraction at curved interface (arbitrary incidence, in the plane of incidence) has the form of

$$M=\begin{bmatrix} 1 & 0 \\ (n_2\cos\theta_2-n_1\cos\theta_1)/R & 1 \end{bmatrix}. \tag{8}$$

As comparison, the same optical experiment was carried out to prove the accuracy of Eq. (8). As shown in Fig. 1(b), the position of incidence ray L_i emitted from spherical center can be specified as the ray vector $[\,x_i \quad n_i\sin\theta_i\,]^T=[\,R\sin\theta_{10} \quad n_1\sin\theta_{10}'\,]^T$ with respect to the z_i axis in the coordinate system (x_i, y_i, z_i) in Fig. 1(b) and the position of experimental refracted ray L_t can be expressed as ray vector $[\,x_t \quad n_2\sin\theta_t\,]^T=[\,R\sin\theta_{10} \quad n_2\sin\theta_{10}'\,]^T$ with respect to the z_t axis in the coordinate system (x_t, y_t, z_t) in Fig. 1(b). According to the definitions of θ_1 and θ_2 in Fig. 1(b) in this paper, it is worthwhile to note that the incident angle and refracted angle of L_i at curved surface are both 0 ($\theta_1=\theta_2=0$) when L_i is perpendicularly incident at the curved surface. The theoretical refracted beam L_t' of L_i at the curved interface can also be obtained with the ray matrix