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A novel hyperchaotic system and its circuit implementation

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ABSTRACT

A four-dimensional hyperchaotic system with five parameters is proposed. Its dynamical properties such as dissipativity, equilibrium points, Lyapunov exponent, Lyapunov dimension, bifurcation diagrams and Poincare maps are analyzed theoretically and numerically. Theoretical analyses and simulation tests indicate that the new system's dynamics behavior can be periodic attractor, chaotic attractor and hyperchaotic attractor as the parameter varies. Finally, the circuit of this new hyperchaotic system is designed and realized by Multisim software. The simulation results confirm that the chaotic system is different from the existing chaotic systems and is a novel hyperchaotic system. The system is recommendable for many engineering applications such as information processing, cryptology, secure communications, etc.

1. Introduction

In the last three decades, chaos has been intensively investigated within the electronics, informatics, mathematics, physics and engineering science, etc. Construct new chaotic system and study its dynamics are useful to explore the nature of chaotic signal and expand the scope of chaos research [1-6]. Hyperchaotic system, which has at least two positive Lyapunov exponents, has more complex dynamic behaviors and can be more useful in various scientific and engineering applications. It is very important to generate hyperchaos with more complicated dynamics as a model for theoretical research and practical implication.

Since the hyperchaos was firstly reported by Rossler in 1979 [7], and the first circuit implementation of hyperchaos was realized by Matsumoto et al. [8], many hyperchaotic systems had been reported. Currently, the main methods to build a new hyperchaotic system are: (1) designing a hyperchaotic system by changing an originally chaotic but non-hyperchaotic system [9–13]; (2) designing a new hyperchaotic system based on the necessary conditions for hyperchaotic systems [14,15].

This work presents a new hyperchaotic system with larger scope of parameters. The generated hyperchaotic system is not only demonstrated by Lyapunov exponent spectrum, bifurcation analysis and Poincare maps, but also verified with circuit realization. The Multisim results of the hyperchaotic circuit show very good agreement with the simulation results. The proposed system, with rich chaotic dynamics behavior, is very desirable for a variety of engineering applications.

2. A new hyperchaotic attractor

The two necessary conditions to generate hyperchaotic system are: (1) For autonomous system, it is at least four-dimensional; (2) it has at least two positive Lyapunov exponents and the sum of all Lyapunov exponents is less than zero. Based on the above two necessary conditions, a new four-dimensional hyperchaotic system is established by the following equations:

$$\begin{aligned} \dot{x} &= a(y-x)+3.5w \\ \dot{y} &= dx-cy-xz^2+3.5w \\ \dot{z} &= -bz+(x+y)x \\ \dot{w} &= rw-\tfrac{1}{3}(x+y)z \end{aligned} \tag{1}$$

where x, y, z, w are state variables and a, b, c, d, r are system parameters.

When $a=17.5$, $b=1.5$, $c=5$, $d=43$, $r=0.5$, Lyapunov exponents of the system (1) are calculated as: $LE_1=0.807587$, $LE_2=0.289090$, $LE_3=-0.009274$, $LE_4=-24.277757$. For LE_1, $LE_2>0$, $LE_1+LE_2+LE_3+LE_4=-23.190354<0$, the four-dimensional system has two positive Lyapunov exponents and the sum of all Lyapunov exponents is less than zero. The new system is a hyperchaotic system. The corresponding hyperchaotic attractor is depicted in Fig. 1.

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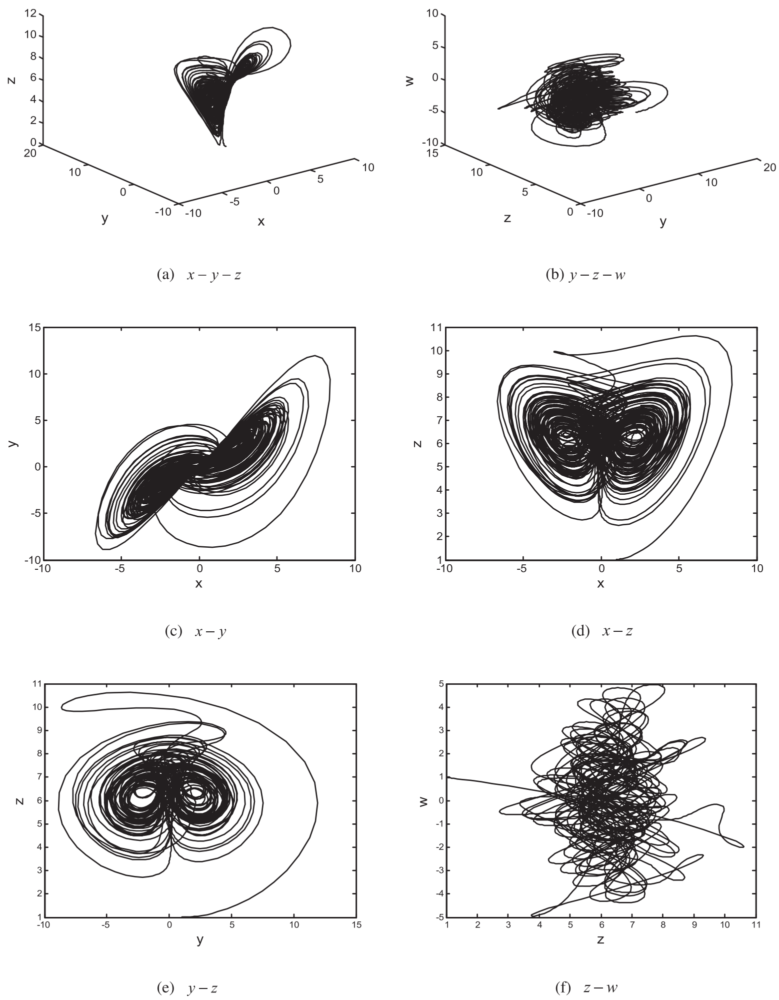

(a) $x-y-z$ (b) $y-z-w$

(c) $x-y$ (d) $x-z$

(e) $y-z$ (f) $z-w$

Fig. 1. Phase diagrams ($a = 17.5$, $b = 1.5$, $c = 5$, $d = 43$, $r = 0.5$).

The Lyapunov dimension of the system (1), a quantity of fractal dimension of a chaos attractor, is described as

$$\begin{aligned} D_{\mathrm{L}} &= \mathrm{j} + \frac{1}{|\lambda_{j+1}|}\sum_{j=1}^{j}\lambda_j = 2 + \frac{\lambda_1+\lambda_2}{\lambda_3+\lambda_4} \\ &= 2 + \frac{0.807587+0.289090}{|-0.009274-24.277757|} = 2.04515 \end{aligned} \tag{2}$$

3. Dynamical analysis of the new hyperchaotic attractor

3.1. Dissipativity and the existence of attractor

For system (1), the divergence of flow of the system is examined as

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -a - c - b + r \tag{3}$$

In order to ensure the dissipation of the system, it is required that $-a - c - b + r < 0$.

An initial volume element $V(0)$ shrinks exponentially by the flow to a volume element $V_0\,\mathrm{e}^{-(a+c+b-r)t}$ as time goes. That is, each volume containing the system trajectory becomes zero as $t \to \infty$. Every trajectory is eventually confined to a specific zero-volume limit set and the asymptotic motion settles onto an attractor of the system (1). When $a = 17.5$, $b = 1.5$, $c = 5$, $d = 43$, $r = 0.5$, the system converges at a rate of $\mathrm{e}^{-23.5t}$. Therefore, each volume cell containing the trajectories of the system ultimately shrinks into zero as $t \to \infty$.

3.2. Equilibrium points and stability

For calculating equilibrium point, let

$$\begin{aligned} &a(y-x) + 3.5w = 0 \\ &dx - cy - xz^2 + 3.5w = 0 \\ &-bz + (x+y)x = 0 \\ &rw - \tfrac{1}{3}(x+y)z = 0 \end{aligned} \tag{4}$$

Eq. (4) leads to the only one equilibrium point $S_0(0, 0, 0)$. Based on this equilibrium point, we investigate stability of the system (1). To this end, we use the method of linearization. The system (1) is linearized at the equilibrium point $S_0(0, 0, 0)$ to obtain the Jacobian matrix as follows:

$$J_0 = \begin{bmatrix} -a & a & 0 & 3.5 \\ d & -c & 0 & 3.5 \\ 0 & 0 & -b & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \tag{5}$$