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Specially profiled reflector for spherical front beams focusing

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ABSTRACT

Focusing the power emitted from a source with spherical wave front by a specially profiled reflecting surface has been attempted using the simple ray tracing method. The reflector (mirror) surface profile that achieves such power redistribution is obtained from solving an ordinary first order differential equation using common numerical techniques. Furthermore, the proposed methodology can be useful to design complex reflectors for the electomagnatic and the acoustic systems, which finds broad engineering, industrial and biomedical applications.

1. Introduction

Reflectors are very used in focusing or redistributing of the electromagnetic and acoustic waves. Focusing waves have many engineering applications; for instance, the focusing reflectors can be used for solar concentrators; communications broadcasting and receiving antenna; industrial machining, welding and undesired tissues abolition by a properly focused laser beam. The front of the propagating waves can be either a plane or a spherical one, depending on the emitting source and the configuration of the system that employs the focusing reflectors. Usually, the spherical front waves are accounted as plane front ones when the working distance between the emitting source and the reflector is practically large; this aspect, to some degree, is applied to simplify the reflector design. It is well known that the parabolic reflecting surface is the choice for focusing the plane front waves; however, the emitted waves from practical emitters, like light emitting diodes and piezoelectric acoustic sources, that are used for relatively short distances application are better be treated as a spherical front ones. Even for laser applications, like industrial welding and machining or for tissue burning and abolition, the spatially diverging laser beam has a radial propagating off the source, therefore, it has spherical front.

Many design methods [1–5] has been developed to synthesis a proper reflector that meets the application requirements; nevertheless, some of the known methods are highly complicated and depends on optimization methods [2], which means the perfect reflector profile is not realized, but instead an optimal one depending on the defined merit function. In this work the focusing of spherical front propagating waves from isotropic like emitting source by a specially profiled reflecting surface will be attempted using the simple ray tracing method. The reflector (mirror) surface profile that achieves such task was found from solving an ordinary first order differential equation using common numerical techniques like the general Runge–Kutta method. The designed reflectors can be useful in broad applications like realizing effective laser beam focusing for industrial machining and surgical purposes, or the focusing of acoustic waves that are used for imaging and/or abolition of tissues for biomedical applications.

2. Theory

It is well known that parabolic reflector are used to focus a plane front beam that incident parallel to the optic axis of the reflector, as shown in Fig. 1(a). Nevertheless, the problem here is rather more general, which is to find the profile $y(x)$ of a special reflector (mirror) that could focus the radial emitting rays originating from point $(0,R)$ at point $(0,F)$ on the optic axis of the reflector as shown in Fig. 1(b).

For the sake of comparison and taking the problem gradually toward generalization, we start with the derivation of the parabolic reflector profile that could focus a plane front beam striking the reflector parallel to optic as shown in Fig. 1(a). Referring to Fig. 1(a), we can write:

$$\tan\,\alpha = \frac{1}{\tan 2\beta} \tag{1}$$

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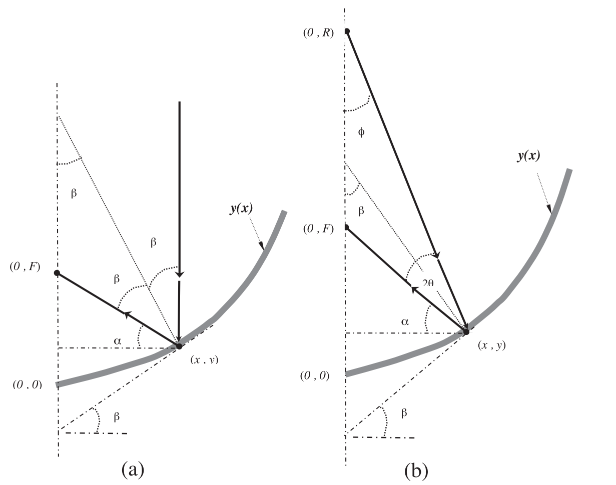

Fig. 1. (a) Ray tracing for parabolic reflector focusing of plane front beam. (b) Ray tracing for proposed reflector focusing of spherical front beam.

Substituting $\tan\alpha = F - y/x$ and $\tan 2\beta = 2y'/\left(1 - y'^2\right)$ into Eq. (1) yields:

$$F = y + x\left(\frac{1 - y'^2}{2y'}\right) \tag{2}$$

To achieve focusing we set $\mathrm{d}F/\mathrm{d}x = 0$, this leads to the following differential equation:

$$y'' = \frac{y'}{x} \tag{3}$$

Eq. (3) has the following general solution:

$$y(x) = ax^2 + b \tag{4}$$

However, substituting $y(0) = 0$ yields $b = 0$; consequently, substituting $y(x) = ax^2$ in Eq. (2) leads to:

$$y(x) = \frac{x^2}{4F} \tag{5}$$

Eq. (5) presents the well known parabolic surface.

Next we consider the more general case, which is the main objective of this work; that is, when the rays originate from a point source located at distance R on the optic axis. Note that as R becomes very large (i.e. $R \to \infty$) the rays turn to be parallel to the optic axis, which indeed presents the first case that has been treated above.

Referring to Fig. 1(b) we could summarize the following set of angels relations:

$$\phi + \theta = \beta \tag{6}$$

$$\phi + 2\theta = \frac{\pi}{2} - \alpha \tag{7}$$

Eliminating θ in the two equations leads to:

$$\alpha - \phi = \frac{\pi}{2} - 2\beta \tag{8}$$

Taking the tangent of both sides of Eq. (8) yields:

$$\frac{\tan\alpha - \tan\phi}{1 + \tan\phi\tan\alpha} = \frac{1}{\tan(2\beta)} \tag{9}$$

Additionally, from Fig. 1(b) the following angles' tangents can be obtained:

$$\tan\alpha = \frac{F - y}{x} \tag{10}$$

$$\tan\phi = \frac{x}{R - y} \tag{11}$$

$$\tan\beta = y' \tag{12}$$

Substituting Eqs. (10)–(12) in Eq. (9) yields:

$$\frac{1 - y'^2}{2y'} = \frac{RF - (R + F)y + y^2 - x^2}{(R + F)x - 2xy} \tag{13}$$

Assuming the right side of Eq. (13) as:

$$g(x, y) = \frac{RF - (R + F)y + y^2 - x^2}{(R + F)x - 2xy} \tag{14}$$

Eq. (13) is a quadratic equation for y' that has a useful solution $y' \geq 0$ as:

$$y' = -g(x, y) + \sqrt{g(x, y)^2 + 1} \tag{15}$$

Eq. (15) is a first order ordinary differential equation that could be solved numerically using the initial condition $y(0) = 0$.

For the case when the reflector is required to focus plane front beams we simply set $R \to \infty$, this reduces Eq. (13) to:

$$\frac{1 - y'^2}{2y'} = \frac{F - y}{x} \tag{16}$$

Obviously, Eq. (16) is similar to Eq. (2), which represents a differential equation with the quadratic profile solution that is given in Eq. (5).

Interestingly, the above theory and results can also be applied for the design of convex reflectors. Such reflectors can be used to control the special illumination a semi-isotropic emitting source as will be demonstrated in the following application examples section. It is not hard to show in similar derivation procedure that the same results are obtained but with a negative value for the distance F in Eq. (14).

3. Application examples

In this section two applications are considered in order to provide a better understanding about the usefulness of the proposed reflectors.

Example1. The first application is to employ a concave reflector with a laser pointer in order to control the spatial divergence of the laser emission. Usually, quality laser pointers that exhibit a tolerated beam divergence are rather expensive devices since they incorporate complicated technological components like Bragg optical filters [6], which indeed increases the fabrication cost of the laser diode dramatically. However, using a regular cheap laser diode with the proposed concave reflector as depicted in Fig. 2(a) could control the divergence of the laser pointer and achieve the desired aiming accuracy. For instance, we consider a laser diode with beam divergence as 10 mrad and it is required to aim at a target that is 1 km far with spatial resolution accuracy 1 cm. It is not hard to show that aiming with the given laser diode yield a beam size of radius 500 cm which is far from being useful for the desired accuracy; besides the fact that the laser power intensity is spread over a far wider area at the target side that it would be hard to detect. In order to realize the aiming accuracy we shall use a concave reflector that is positioned at a practical distance like 50 cm in order to control the divergence of the pointing beam; therefore, the used F is 50 cm, and the radius of the laser beam at the reflector side is about 5 mm; consequently, the converging angle after the reflector can be estimated by:

$$\theta_{\mathrm{conv}} \approx \frac{5\,\mathrm{cm} - 1\,\mathrm{cm}}{10^5\,\mathrm{cm}} = 0.04\,\mathrm{mrad}.$$

Therefore, the distance R is estimated as follow:

$$R \approx \frac{5\,\mathrm{cm}}{\theta_{\mathrm{conv}}} = 1.2\,\mathrm{km}.$$

From the defined values of F and R the function $g(x,y)$ can be determined using Eq. (14); therefore, the differential Eq. (15) is