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Effects of low order atmosphere-turbulence aberrations on the vortex carrying Gaussian beam

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ABSTRACT

The normalized intensity distributions at the focal plane of a vortex carrying Gaussian beam propagating in weak/middle turbulent atmosphere channel with z-tilt aberration, defocus aberration, astigmatism aberration or total turbulent aberration are discussed by numerical calculation. Our results show that the effect of z-tilt aberration on the intensity distribution of optical vortex beam is main effect of total turbulent aberration. In weak turbulent region, the effects of defocus and astigmatism aberration on the intensity distribution of a vortex carrying Gaussian beam can be ignored. In middle turbulent region, the effect of z-tilt aberration is still the most significant, but the effects of defocus and astigmatism aberrations, specially the effect of astigmatism on the center dark core of the intensity distribution at focal plane, can no longer be disregarded. Our results also show that for three low order aberrations, the beam with values of the topological charge, the beam have larger beam-radius and undergo smaller effects of z-tilt turbulent aberration on the doughnut distribution. For defocus aberration, the radius of the center dull of beam intensity increases with the values of topological charges increasing. And for astigmatism aberration, the beam with odd number topological charges, the center dull speck of beam intensity becomes to the bright speck. But for even number topological charges, the center dull speck of beam intensity maintains the dull one.

1. Introduction

Because of the wide application in space optical communication, high resolution imaging and optical remote sensing, the behaviors of the optical beams propagating in turbulent atmosphere have generated considerable interest in many researchers [1–6]. Due to the special propagation properties of the vortex beams propagation through the apertures system in the presence of aberrations of the turbulent atmosphere, much work has been done to investigate the propagation and diffraction performance of the beam with optical vortices. Rakesh et al. investigated the focusing vortex beam with Gaussian background by an apertures system in the presence of coma [7], spherical aberration [8], astigmatism and defocusing [8,9]. Zhang et al. [10] discussed the propagation property of a partially coherent vortex beams propagating in the turbulent atmosphere. Zhao et al. [11,12] studied the effect of tilt, defocus and astigmatism caused by turbulent atmosphere on the intensity distribution of a focused vortex carrying beam with a Gaussian background. Gbur and Tyson [13] simulated and analyzed the conservation features of topological charge of the propagation of vortex beams through weak-to-strong atmospheric turbulence. Wang et al. [14] theoretically and experimentally investigated the beam-spreading of a vortex beam propagating in a turbulent atmosphere and found that the vortex beam is less affected by turbulence than an no-vortex one. Aksenov [15] studied the fluctuation of orbital angular momentum of vortex laser-beam in turbulent atmosphere. Tyler and Boyd [16] analyzed the influence of atmospheric turbulence on the propagation of quantum states of light carrying orbital angular momentum. As is well known, in order to improve the imaging resolution and enhance the quality of the beams by means of optical adaptive technique, we need to know the modulating effect of every turbulent aberration on the optical beams.

In this paper, the effects of Zernike-turbulence tilt aberration, turbulent astigmatism, turbulent defocus and total aberration of turbulent atmosphere on the intensity distribution of a focused vortex carrying beam with a Gaussian background are studied by the numerical calculation approached. The paper is organized as follows: the effect the total turbulent aberration on the intensity distribution of the vortex carrying Gaussion beams is given in Section 2. In Section 3, the intensity distributions of beam propagating in channel with Zernik-turbulence tilt aberration, turbulent astigmatism and turbulent defocus aberration are derived. Some numerical calculations and analysis related to beam propagation in free space channel or in channel with Zernik-turbulence tilt aber-

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ration, turbulent astigmatism and turbulent defocus aberration are given in Section 4. A summary is presented in the conclusion.

2. Effects of total turbulent aberrations on the intensity distribution

The complex amplitude at the observation plane of a beam propagating in the turbulent atmosphere can be evaluated by using the Fresnel–Kirchhoff diffraction integral [7–9] and turbulent aberration $S(\rho,\theta)$, which can be written, at the focal plane, as

$$E(r,\varphi,z)=C_1\int_0^1\int_0^{2\pi}E(\rho,\theta,z=0)\exp\{iS(\rho,\theta)\}\exp\times\left[-i\frac{2\pi a}{\lambda z}r\rho\cos(\theta-\varphi)\right]\rho\,d\rho\,d\theta \quad (1)$$

where $C_1=a^2\exp(ikz+i\pi r^2/\lambda z)/i\lambda z$, $S(\rho,\theta)$ is the wave front aberration function of the turbulent atmosphere in the polar coordinates (ρ,θ) which can be decomposed into Zernike polynomials [17,18], $k=2\pi/\lambda$ is the optical wave number, λ is wavelength, z is the propagation distance, a is radius of exit pupil, $E(\rho,\theta,z=0)=E_0(\sqrt{2}\gamma\rho)^{|m|}\exp(-\gamma^2\rho^2+im\theta)$ is the complex amplitude of a beam with vortex at the origin of the transverse plane embedded in the Gaussian background. E_0 is the characteristic amplitude, ρ is the radial distance of a point from its center normalized by radius a, θ is the azimuthally coordinate on the $z=0$ plane, m is topological charge and $\gamma=(a/w_0)$ is a truncation parameter with w_0 (waist of beam) as a parameter for beam size at the $z=0$ plane. The beam waist w_0 together with E_0 scales the peak intensity of the beam. The factor C_1 does not affect the functional form of the intensity distribution and hence ignored. Intensity distribution $I(r,\phi,z)$ at the focal plane of a focused vortex carrying Gaussian beam is given by

$$I(r,\phi,z)=\int_0^1\int_0^{2\pi}\int_0^1\int_0^{2\pi}(2\gamma^2\rho_1\rho_2)^{|m|}\exp[-\gamma^2(\rho_1^2+\rho_1^2)]\exp[im(\theta_1-\theta_2)]\times\exp\left[-\frac{1}{2}D_s(\bar{\rho}_1,\bar{\rho}_2)\right]\exp\left\{i\pi\left(\frac{2a}{\lambda z}\right)r[\rho_2\cos(\theta_2-\phi)-\rho_1\cos(\theta_1-\phi)]\right\}\rho_1\rho_2\,d\rho_1\,d\theta_1\,d\rho_2\,d\theta_2 \quad (2)$$

here $D_s(\bar{\rho}_1,\bar{\rho}_2)$ is the phase structure function [19]. For the Kolmogorov power-law spectrum, the structure function $D_s(\bar{\rho}_1,\bar{\rho}_2)$ of the total turbulent aberration can be written as [18]

$$D_s(\bar{\rho}_1,\bar{\rho}_2)=6.882^{-5/3}\left(\frac{D}{r_0}\right)^{5/3}(\bar{\rho}_1-\bar{\rho}_2)^{5/3} \quad (3)$$

here $r_0\cong\left(0.432k^2\sec\varsigma\int_0^H C_n^2(h)\,dh\right)^{-3/5}$ is the coherent length [19] of the spherical wave propagating in slant atmospheric-turbulence-path, D is the wave front sampling circular aperture diameter, and ς is zenith angle, $C_n^2(h)$ is refractive index structure constant, which could be calculated as a function of the altitude using Hufnagel–Valley (H–Y) model [19] as follows:

$$C_n^2(h)=0.00594\left(\frac{v}{27}\right)^2\left(\frac{h}{10^5}\right)^{10}\exp\left(-\frac{h}{1000}\right)+2.7\times10^{-16}\exp\left(-\frac{h}{1500}\right)+A\exp\left(-\frac{h}{100}\right) \quad (4)$$

where $h=z\cos\varsigma$ is height (m), $v=21$ m/s is rms wind velocity(m/s), $A=C_n^2(0)(\mathrm{m}^{-2/3})$.

3. Effects of three low-order turbulence aberrations on the intensity distribution

The wave front aberration functions of turbulence z-tilt, defocus and astigmatism can be expressed as [16,17]:

$$S_{\mathrm{tilt}}(\rho,\theta)=2a_2\rho\cos\theta+2a_3\rho\sin\theta\quad \text{z-tilt} \quad (5)$$

$$S_{\mathrm{defocus}}(\rho,\theta)=\sqrt{3}a_4(2\rho^2-1)\quad \text{Defocus} \quad (6)$$

$$S_{\mathrm{asti}}(\rho,\theta)=\sqrt{6}(a_5\rho^2\cos2\theta+a_6\rho^2\sin2\theta)\quad \text{Astigmatism} \quad (7)$$

where a_2, a_3, a_4, a_5, a_6 are the expansion coefficients of Zernike polynomial. The phase construction functions of the three aberrations according to the mean-square phase difference and the quadratic approximation of phase construction function [19] are

$$D_{\mathrm{tilt}}(\bar{\rho}_1,\bar{\rho}_2)=4\langle a^2_{\{2,3\}}\rangle[\rho_1^2+\rho_2^2-2\rho_1\rho_2\cos(\theta_1-\theta_2)]\quad \text{z-tilt} \quad (8)$$

$$D_{\mathrm{defocus}}(\bar{\rho}_1,\bar{\rho}_2)=3\langle a_4^2\rangle(2\rho_1^2-2\rho_2^2)^2\quad \text{Defocus} \quad (9)$$

$$D_{\mathrm{asti}}(\bar{\rho}_1,\bar{\rho}_2)=6\langle a^2_{\{5,6\}}\rangle[\rho_1^4+\rho_2^4-2\rho_1^2\rho_2^2\cos(2\theta_1-2\theta_2)]\quad \text{Astigmatism} \quad (10)$$

here [16,17].

$$\langle a^2_{\{2,3\}}\rangle=0.449\left(\frac{D}{r_0}\right)^{5/3} \quad (11)$$

$$\langle a_4^2\rangle=\langle a^2_{\{5,6\}}\rangle=0.023\left(\frac{D}{r_0}\right)^{5/3} \quad (12)$$

Use Eqs. (8)–(10) instead of $D_s(\bar{\rho}_1,\bar{\rho}_2)$ in Eq. (2), we can obtain the average intensity distribution at the focal plane of the Gaussian beams with the vortices under the effects of the three low order turbulent aberrations

$$I(r,\phi,z)=\int_0^1\int_0^{2\pi}\int_0^1\int_0^{2\pi}(2\gamma^2\rho_1\rho_2)^{|m|}\exp[-\gamma^2(\rho_1^2+\rho_2^2)]\exp[im(\theta_1-\theta_2)]\times\exp\left[-\frac{1}{2}D_{\mathrm{tilt,defocus,asti}}\right]exp\{i\pi\left(\frac{2a}{\lambda z}\right)r[\rho_2\cos(\theta_2-\phi)-\rho_1\cos(\theta_1-\phi)]\}\rho_1\rho_2\,d\rho_1\,d\theta_1\,d\rho_2\,d\theta_2. \quad (13)$$

4. Numerical calculation and discussion

Analytical solution of Eq. (13) is not possible for arbitrary phase structure function of turbulent aberration, and hence a numerical approach is used for the evaluation of the integral. Intensity distributions at the focal plane are calculated by using Eqs. (2) and (13) for beam propagating in horizontal path $\varsigma=90^\circ$ under the influences of z-tilt, defocus, astigmatism aberration and total turbulent aberration, in which the beam propagate through $z=f=2000$ m or 5000 m. Because of the symmetry of the vortex beams, the two-dimensional normalized averaged intensity distributions profile is adopted in this paper. The beam source wavelength is $\lambda=1060$ nm and $a=0.05$ m.

We show the normalized intensity distributions at the focal plane in Figs. 1 and 2 as a function of beam radius for beams, which have topological charge $m=1$ or $m=2$, propagation in free space $C_n^2(0)=0\,\mathrm{m}^{-2/3}$ and in weak/middle turbulent channel ($C_n^2(0)=10^{-15}\,\mathrm{m}^{-2/3}/C_n^2(0)=10^{-14}\,\mathrm{m}^{-2/3}$) with defocus, astigmatism, z-tilt and total turbulent aberration, respectively.

Results in Fig. 1a–d are shown that the effects of defocus and astigmatism on the intensity distribution are very small, for a