Optik 126 (2015) 2264–2268

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Analytical populations of a multilevel atom in a weak linearly-polarized light

Heung-Ryoul Noh*

Department of Physics, Chonnam National University, Gwangju 500-757, Republic of Korea

ARTICLE INFO

Article history:
Received 16 April 2014
Accepted 25 May 2015

Keywords:
Optical pumping
Analytical populations
Rate equations

ABSTRACT

We present analytical solutions of the populations of an alkali-metal atom in the presence of a weak linearly-polarized laser beam. After a general method for calculating rate equations is described, the analytical solutions of the populations for the transition $F_g = 1 \rightarrow F_e = 0, 1, 2$ are obtained. Provided that the laser intensity is weak, the time-dependent analytical populations are exact. As examples, explicit results for the D_2 and D_1 transitions of an alkali-metal atom with a nuclear-spin angular momentum of $I = 3/2$, such as ^{87}Rb, ^{23}Na, ^{7}Li, or ^{39}K, are presented.

1. Introduction

After the advent of the laser, there has been considerable interest in manipulating the internal or external degree of freedom of atoms using laser light [1]. The former includes optical pumping [2], high-resolution laser spectroscopy [3], coherent population trapping (CPT) [4], and electromagnetic-induced transparency (EIT) [5], and the latter includes laser cooling and trapping of atoms, atom optics, and Bose–Einstein condensation of atoms [1]. Of these, optical pumping, which was studied before the introduction of the laser, is now widely used in atomic state preparation of interest [2,6,7]. While accurate analytical solutions can be obtained for two-level atoms [8], numerical studies using the rate equations [9] or a density-matrix formalism [10,11] have been reported for multilevel atoms.

For the analytical solutions of optical pumping for multilevel atoms, the analytical solutions of steady-state populations using a linearly [12] or an elliptically polarized laser light [13] have been reported. We reported the analytical solutions of the populations of the D_1 and D_2 transition lines of ^{87}Rb atoms [14,15], and used the results to calculate the saturated absorption spectra analytically [16]. We also obtained the exact analytical form of the populations for ^{87}Rb atoms using a weak σ^+ polarized laser beam [17] considering all the sublevels in the excited state. Extending previous studies, we report on the analytical calculation of populations under the influence of a weak π polarized laser light, independent of the magnitude of the energy spacings of the excited state. After establishing the equations, although the calculation is applicable to other transitions with higher angular momentum quantum numbers, we restrict our calculation to the transitions $F_g = 1 \rightarrow F_e = 0, 1, 2$ ($F_e = 1, 2$) in the D_2 (D_1) line of an alkali-metal atom, such as ^{87}Rb, ^{23}Na, ^{7}Li, or ^{39}K, with a nuclear-spin angular momentum of $I = 3/2$ [1]. This is because the analytical calculation for other transitions is too difficult to handle analytically. However, exact populations can be obtained by using numerical coefficients rather than an analytical form. Since the method developed in this paper can be applied, regardless of the energy spacings, it would be more useful for atoms with relatively smaller excited-state energy spacings, such as ^{23}Na, ^{7}Li, or ^{39}K.

2. General method of calculation

The energy level diagram of the alkali-metal atom under consideration is shown in Fig. 1(a). The ground state has two hyperfine states ($F_g = F$ and F'). A linearly-polarized single mode laser beam is tuned at the transition $F_g = F \rightarrow F_e = F - 1, F, F + 1$. We assume that the energy spacing between two hyperfine states of the ground state is much larger than the natural linewidth (~10 MHz), so that the populations at the state $F_g = F'$ are not excited by the laser. The rate equations for the excited states are given by [16,17]

$$\begin{aligned} \dot{g}_i &= (\Gamma/2) R_{F,i}^{F+1,i} \eta_{F+1} (f_i - g_i) - \Gamma g_i, \\ \dot{p}_i &= (\Gamma/2) R_{F,i}^{F,i} \eta_F (f_i - p_i) - \Gamma p_i, \\ \dot{q}_i &= (\Gamma/2) R_{F,i}^{F-1,i} \eta_{F-1} (f_i - q_i) - \Gamma q_i, \end{aligned} \tag{1}$$

* Tel.: +82 625303366.
E-mail address: hrnoh@chonnam.ac.kr

http://dx.doi.org/10.1016/j.ijleo.2015.05.120
0030-4026/

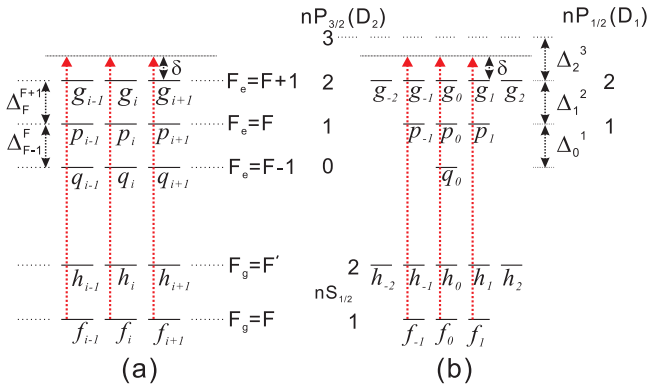

Fig. 1. (a) General energy level diagram. (b) Energy level diagram for the D_2 and D_1 lines of the atoms with $I = 3/2$.

where Γ is the decay rate of the excited states and i denotes the magnetic quantum number. In Eq. (1), the saturation parameters are given by

$$\eta_{F_e} = \frac{\Omega^2/2}{\left(\delta + \Delta_{F_e}^{F+1}\right)^2 + \Gamma^2/4}, \tag{2}$$

where $F_e = F-1$, F, $F+1$, Ω is the Rabi frequency, $\hbar\Delta_{F_a}^{F_b} = E_{F_b} - E_{F_a}$ is the hyperfine energy spacing between excited states, and δ is the detuning of the laser frequency relative to the transition $F_g = F \rightarrow F_e = F+1$. In Eq. (1), the relative transition strength is given by [18]

$$R_{F_g,m_g}^{F_e,m_e} = (2L_e+1)(2J_e+1)(2J_g+1)(2F_e+1)(2F_g+1)$$
$$\left[\begin{Bmatrix} L_e & J_e & S \\ J_g & L_g & 1 \end{Bmatrix} \begin{Bmatrix} J_e & F_e & I \\ F_g & J_g & 1 \end{Bmatrix} \begin{pmatrix} F_g & 1 & F_e \\ m_g & m_e - m_g & -m_e \end{pmatrix}\right]^2,$$

where L, S, and I are the orbital, electron spin, and nuclear spin angular momenta, respectively, and $\{\}$ $[()]$ denotes the $6J$ $[3J]$ symbol.

When the intensity is weak, the populations of the excited states are smaller than those of the ground states by a factor of $\sim(\Omega/\Gamma)^2$. Thus, we can obtain the following equations from Eq. (1):

$$f_i \simeq \frac{2}{\Gamma\eta_{F+1}R_{F,i}^{F+1,i}}(\dot{g}_i + \Gamma g_i) \simeq \frac{2}{\Gamma\eta_F R_{F,i}^{F,i}}(\dot{p}_i + \Gamma p_i)$$
$$\simeq \frac{2}{\Gamma\eta_{F-1}R_{F,i}^{F-1,i}}(\dot{q}_i + \Gamma q_i). \tag{3}$$

Inserting the trial solutions in the form of $\sim e^{\lambda\Gamma t}$ in Eq. (3), we obtain the following relations of proportionality:

$$p_i \simeq \frac{\eta_F R_{F,i}^{F,i}}{\eta_{F+1}R_{F,i}^{F+1,i}} g_i, \quad q_i \simeq \frac{\eta_{F-1}R_{F,i}^{F-1,i}}{\eta_{F+1}R_{F,i}^{F+1,i}} g_i. \tag{4}$$

The rate equation for the ground state sublevel f_i is given by

$$\dot{f}_i = \sum_{\nu=F-1}^{F+1}\left[-\frac{\Gamma}{2}R_{F,m}^{\nu,m}\eta_\nu\left(f_i - \xi_i^{(\nu)}\right) + \sum_{m_e=i-1}^{i+1}\Gamma R_{F,i}^{\nu,m_e}\xi_{m_e}^{(\nu)}\right], \tag{5}$$

where $\xi_i^{(F+1)} \equiv g_i$, $\xi_i^{(F)} \equiv p_i$, and $\xi_i^{(F-1)} \equiv q_i$. If Eqs. (1) and (3) are inserted into Eq. (5), we obtain

$$\frac{2}{\Gamma\eta_{F+1}R_{F,i}^{F+1,i}}(\ddot{g}_i + \Gamma\dot{g}_i) = -\dot{g}_i - \Gamma g_i - \dot{p}_i - \Gamma p_i - \dot{q}_i - \Gamma q_i$$
$$+ \sum_{m_e=i-1}^{i+1}\Gamma\left(R_{F,i}^{F+1,m_e}g_i + R_{F,i}^{F,m_e}p_i + R_{F,i}^{F-1,m_e}q_i\right), \tag{6}$$

Inserting the trial solutions in the form of $\sim e^{\lambda\Gamma t}$ into Eq. (6) and using Eq. (4), we have linear equations for g_i,

$$-(2\lambda + a_i)(1+\lambda)g_i + \sum_{j=i-1}^{i+1} b_{ij}g_j = 0, \tag{7}$$

where

$$a_i = \sum_{\nu=F-1}^{F+1}\eta_\nu R_{F,i}^{\nu,i}, \quad b_{ij} = \frac{R_{F,i}^{F+1,i}}{R_{F,j}^{F+1,j}}\sum_{\nu=F-1}^{F+1}\eta_\nu R_{F,i}^{\nu,j}R_{F,j}^{\nu,j}. \tag{8}$$

Eq. (7) can be expressed as a matrix form, $Mg^T = 0$, where $g = (g_0, g_1, g_2, \ldots, g_{F-1}, g_F)$ and M is given by

$$M = \begin{pmatrix} Q_0 & 2b_{01} & 0 & 0 & \cdots & 0 & 0 & 0 \\ b_{10} & Q_1 & b_{12} & 0 & \cdots & 0 & 0 & 0 \\ 0 & b_{21} & Q_2 & b_{23} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & b_{F-1,F-1} & Q_{F-1} & b_{F-1,F} \\ 0 & 0 & 0 & 0 & \cdots & 0 & b_{F,F-1} & Q_F \end{pmatrix},$$

with $Q_i = b_{ii} - (2\lambda + a_i)(1+\lambda)$. It should be noted that, since the π polarized light is used, the populations have a symmetric property, i.e., $f_{-i} = f_i$, $g_{-i} = g_i$, $p_{-i} = p_i$, $q_{-i} = q_i$, and $h_{-i} = h_i$ for all relevant i. λ can be calculated by equating the secular equation of the matrix M. The secular equation in the lowest order in the saturation parameters contains the term $\lambda(1+\lambda)$. Therefore, the lowest order term in λ is 0 or -1, and thus λ can be expressed as $\lambda \sim O(\eta_i)$ or $\lambda \sim -1 + O(\eta_i)$, which is called case A and case B, respectively. In case A, $Q_i \simeq Q_i^{\mathrm{A}} = b_{ii} - (2\lambda + a_i)$. In contrast, in case B, $Q_i \simeq Q_i^{\mathrm{B}} = b_{ii} + 2\chi$ where $\chi \equiv \lambda + 1$. In each case, the number of solutions is $F+1$. In the calculation, only the solutions in case A are important.

After the values of λ are determined, g_i is calculated as follows:

$$g_i = \sum_{n=1}^{F+1}u_{i,n}e^{\lambda_n\Gamma t} + \sum_{n=1}^{F+1}v_{i,n}e^{(-1+\chi_n)\Gamma t}, \quad i = 0, \ldots, F, \tag{9}$$

where the first (second) term in the right-hand side represents case A (B). The coefficients in Eq. (9) are expressed in terms of $u_{0,n}$ and $v_{0,n}$ $(n = 1, \ldots, F+1)$ using the relations:

$$u_{i,n} = \frac{(2\lambda_n + a_{i-1} - b_{i-1,i-1})u_{i-1,n} - b_{i-1,i-2}u_{i-2,n}}{b_{i-1,i}}, \quad \text{for } i \geq 2,$$
$$u_{1,n} = \frac{(2\lambda_n + a_0 - b_{0,0})u_{0,n}}{2b_{0,1}},$$
$$v_{i,n} = \frac{-(2\chi_n + b_{i-1,i-1})v_{i-1,n} - b_{i-1,i-2}v_{i-2,n}}{b_{i-1,i}}, \quad \text{for } i \geq 2,$$
$$v_{1,n} = -\frac{(2\chi_n + b_{0,0})v_{0,n}}{2b_{0,1}}.$$

The coefficients $u_{i,n}$ and $v_{i,n}$ in Eq. (9) can be determined from the boundary conditions: $g_i(0) = 0$ and $\dot{g}_i(0) = \Gamma\eta_{F+1}R_{F,i}^{F+1,i}/[4(2I+1)]$ for $i = 0, \ldots, F$, where we used the fact that $f_i(0) = 1/[2(2I+1)]$ is the population of each sublevel in the ground state at equilibrium. When atoms with $I = 3/2$ are considered, $f_i(0) = 1/8$. Once the