Optik 126 (2015) 2362–2367

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Dispersion study on scattering cross section of metamaterial cloak due to various cloaking parameters

Archana Rajput*, Kumar Vaibhav Srivastava

Department of Electrical Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India

ARTICLE INFO

Article history:
Received 20 April 2014
Accepted 29 May 2015

Keywords:
Scattering cross section
Metamaterial cloaking
Invisibility
Transformation optics

ABSTRACT

Electromagnetic cloak is a device that has the potential to conceal an object by minimizing its total scattering cross section over a desired frequency. But due to causality constraints the material properties become dispersive which reduces cloaking bandwidth. In this work, the emphasis is on the study of various cloaking parameters like dispersive material properties, cloak dimensions, material losses which influence the real implementation of cloak. This work rigorously investigated the scattering performance at the design frequency and cloaking bandwidth for aforementioned parameters on the cloak design by utilizing a widely accepted term called total scattering cross section.

1. Introduction

Idea of invisibility has created inquisitiveness among researchers and engaged their interests due to fascinating attributes of invisibility. Transformation optics [1–4] (TO) has enlightened the concept of anisotropic and spatially varying media based on the invariance of Maxwell's equation, which perfectly hides an object by smoothly guiding the electromagnetic (EM) waves around an object without producing any changes in the free space. The real implementation of anisotropic and spatially varying media, commonly not found in nature, is possible with metamterials [5,6].

Primarily, an object is visible due to the scattering of incident EM waves. Thus, the foremost requirement of electromagnetic cloaking is to reduce scattering from an object on EM illumination. To achieve perfect cloaking, the total scattering cross-section [7,8] should be zero which ensures the reduction of scattered field in every direction. The analytical study showing the interactions of EM waves with cloak has been described in [9] based on a full wave Mie scattering model. Scattering performance has been analyzed for tiny perturbation in an ideal cylindrical cloak [10]. Scattering coefficient study in different cylindrical wave orders have been shown with simplified material properties [11,12]. Until now, discussions on the bandwidth and practical limitations of electromagnetic cloak present in the published results [13–18] are few in number.

In the context of the practical implementation, this work investigates the effect of frequency dependent material properties and material losses which restricts the usefulness of cloak by limiting bandwidth. Here, FEM solver based analysis has been carried out to obtain the operational bandwidth and minimum scattering cross section of cloak design for various cloaking parameters. The results show that the bandwidth and cross section are dependent on the type of the material properties used in the cloaking shell. The cloaking performance degrades by using simplified material properties compared to the exact material properties. The investigation on cloak dimensions (outer and inner radii) is done for the various cases of frequency dependent material properties. It is seen that if impedance at the outer boundary of the cloak is matched to the free space impedance, cloaking performance remains same with change in the outer radius. Otherwise, scattering performance is dependent on the outer radius. This work also demonstrates the effect of scatterer size on the cloak design and reveals that the bandwidth of the cloak decreases with increase in the scatterer size irrespective of different cases of material properties applied within the cloaking shell. Losses are introduced as an imaginary part on the material properties. With the increase of the losses, bandwidth increases but at the penalty of increased scattering cross section. Thus, material losses degrade the cloaking performance significantly. The study of dispersive cloaking parameters has been carried out which plays a vital role in cloak's practical implementation.

2. Theoretical analysis

Cloak is a device that scatters very little (ideally zero), and the cloaking is the ability of such devices that can reduce the total

* Corresponding author. Tel.: +91 5122597733.
E-mail addresses: rajput.archana07@gmail.com (A. Rajput), kvs@iitk.ac.in (K.V. Srivastava).

http://dx.doi.org/10.1016/j.ijleo.2015.05.141
0030-4026/

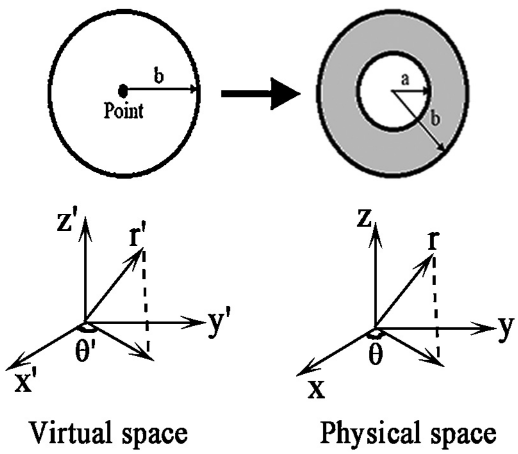

Fig. 1. Cross-sectional view (x–y) of the cylindrical structure for coordinate transformation is illustrated here. A cylindrical region, $r' \in [0, b]$ (virtual space) is transformed into a concentric cylindrical region, $r \in [a, b]$ (physical space) during the coordinate transformation.

scattering cross section of an object from arbitrary incidence. The scattering cross section (SCS) [7], σ can be written as

$$\sigma = \lim_{r\to\infty} 2\pi r \frac{|E_s|^2}{|E_{inc}|^2} \quad (1)$$

where E_s is the scattered field at a far field distance of r and E_{inc} is the incident electric field. The total scattering cross section is obtained by integrating the SCS over all angles. Normalized total scattering cross section, $SCS_{t,norm}$ can be calculated by taking the ratio of total SCS of cloaked object to the total SCS of uncloaked object (uncloaked object: perfect electric conductor). The frequency range over which $SCS_{t,norm}$ is less than 1 determines the cloaking bandwidth [13,19].

To show the numerical analysis, a 2D cylindrical geometry is considered with dimensions a and b corresponding to inner and outer radii of the cloak design, which is illuminated by transverse electric (TE) plane wave. Similar analysis can also be done by transverse magnetic (TM) plane wave. Cross-sectional view of the cylindrical structure is presented in Fig. 1, which shows the coordinate transformation that squeezes a cylindrical region, $r' \in [0, b]$ (virtual space) into a concentric cylindrical region, $r \in [a, b]$ (physical space). The transformation function $r = f(r')$ is given by following relation [1]:

$$r = f(r\prime) = \left(\frac{b-a}{b}\right) r\prime + a \quad (2)$$

which satisfies the boundary conditions $f(0) = a$ and $f(b) = b$, where r' and r are the radial axis in the virtual and physical space. Based on the above transformation three cases of material properties have been considered.

Case I: The expressions of the exact material properties [20] in the cloaking shell are given by

$$\mu_r = \frac{r-a}{r} \quad (3a)$$

$$\mu_\theta = \frac{r}{r-a} \quad (3b)$$

$$\epsilon_z = \left(\frac{b}{b-a}\right)^2 \left(\frac{r-a}{r}\right) \quad (3c)$$

Here, all the material properties have radial dependency, such complex media increases the difficulty level of practical realization. To resolve this, the set of Eq. (3) can be modified [20] in two ways to give the simplified material properties shown by Case II and Case III.

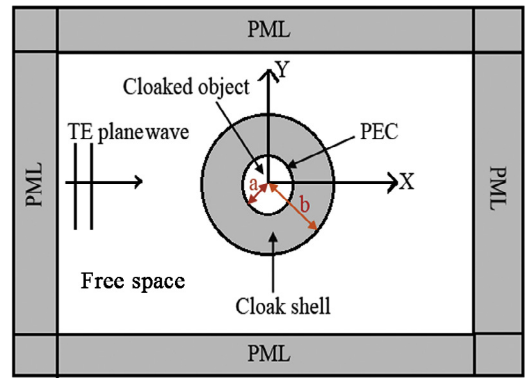

Fig. 2. Modelling of a 2D cylindrical cloaking problem in the computational domain.

Case II: The matched material properties [12] within the cloaking shell are

$$\mu_r = \left(\frac{b}{b-a}\right)\left(\frac{r-a}{r}\right)^2 \quad (4a)$$

$$\mu_\theta = \left(\frac{b}{b-a}\right) \quad (4b)$$

$$\epsilon_z = \left(\frac{b}{b-a}\right) \quad (4c)$$

Case III: The reduced material properties [20] within the cloak designing are

$$\mu_r = \left(\frac{r-a}{r}\right)^2 \quad (5a)$$

$$\mu_\theta = 1 \quad (5b)$$

$$\epsilon_z = \left(\frac{b}{b-a}\right)^2 \quad (5c)$$

The wave impedance z_θ [8,20] for TE polarization is defined as $z_\theta = \sqrt{\mu_\theta/\epsilon_z}$. The effects of simplified material properties on the cloak performance have been presented in Section 3.1. Section 3.2 addresses the effects of cloak dimensions on the cloaking operation using three different cases of material properties.

In this work for cloak design, the frequency dependent permittivity (or permeability) are modelled by using Drude dispersive relation [14]

$$\epsilon = 1 - \frac{{f_p}^2}{f(f + i\gamma)} \quad (6)$$

where f is the working frequency, f_p is the plasma frequency, γ represents damping coefficient. Losses [21,17] are inherent in metamaterials which further restrains the benefits of cloak. So, the effects of losses on the cloak performance have been studied in Section 3.3.

3. Numerical simulation and discussions

To demonstrate the cloaking effect simulations are performed in a commercial finite element software package, COMSOL Multiphysics. In this section, 2D cloaking structure is modelled in XY plane and illuminated by TE plane wave travelling in the +X direction as shown in Fig. 2. The operating frequency is considered as 1.5 GHz. Perfect electric conductor (PEC) is set at the inner boundary of the cloaking shell. Perfectly matched layer (PML) is applied at the outer boundary of the computational domain, which absorbs the scattered field from the interior domain without producing reflections back to it. SCS is calculated at the inner layer of PML boundary.