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Research of correcting distorted X-ray images based on least squares and biharmonic spline surface

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ABSTRACT

Owing to the geometric distortions of the X-ray images, C-arm CT (computed tomography) imaging system usually suffers from adverse effects. To overcome the difficulties, an integrated approach has been put forward to correct distorted X-ray images in the paper. The approach firstly uses OTSU algorithm and morphological open operation to extract the control points coordinates in the distorted X-ray images. To obtain more coordinates and improve the correction precision, least squares is used to fit the polynomial passing by the extracted points. More coordinates are calculated by using the fitted polynomial. Then, a biharmonic spline surface is interpolated by the extracted and calculated coordinates. Finally, the distorted X-ray images are corrected by using the interpolated biharmonic spline surface. Experiment on the acquired distorted X-ray images demonstrates that the proposed approach can effectively correct the distorted X-ray images. A comparison with classical approaches further shows that the approach proposed in the paper performs better in visual effects and correction accuracy.

1. Introduction

Though there are more and more people using the amorphous silicon flat panel detector instead of X-ray image intensifier in the developed countries or areas, in other countries or areas, especially in the developing world, X-ray image intensifier is still widely applied in investigating scatter radiation doses [1], tracker configuration [2] and cerebral blood flow measurements [3], and so on [4]. But because of the curved input surface, manufacturing technology and earth's magnetic field [5,6], some distorted phenomena such as the pincushion distortion, sigmoid distortion [5,6] and local distortion [7] appear in the acquired projection data.

Aimed at the distorted X-ray images brought about adverse influence on C-arm CT imaging system, scholars have published some calibration approaches. These calibration approaches are classified into the global [5–12] and local calibration technique [13]. Owing to discontinuous phenomenon at patch boundary and low correction accuracy at intermediate points, the local calibration technique has been criticized [7]. The global calibration approach can effectively avoid the shortcoming of the local technique. Nevertheless, this technique is sensitive to sigmoid and local distortion [7].

We propose an integrated approach to correct the distorted X-ray images in this paper. The approach consists of least squares and biharmonic spline surface. The technique of least squares has been widely used in many fields: statistics [14], image processing [15] and physics [16,17], and so on [18]. Biharmonic spline surface has been used in some fields, such as marine satellite measurement data [19], integration of logging and seismic data [20], image deformation [21], to name a few [22]. To our knowledge, few researches published the integrated approach with least squares and biharmonic spline surface in correcting distorted X-ray images. The proposed approach has been compared with the classical techniques in the precises and visual effects. Experiment on the acquired images (1024 × 1024 pixels) using C-arm CT imaging system have demonstrated that the technique proposed in this paper can achieve better visual effects and more precises.

2. Least squares and biharmonic spline surface interpolation

2.1. Least squares

Let $(x_i, y_i)(i=0, 1, \ldots, m)$ be a group given points in every row for the projection of X-ray image intensifier. Φ is a set of

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polynomials $\{1, x_i^1, x_i^2, x_i^3, \ldots, x_i^n\}(i = 0, \ldots, m)$. $p_n(x_i)p_n(x_i) = a_0 + a_1x_i + a_2x_i^2 + a_3x_i^3 + \cdots + a_nx_i^n$ is a combination of polynomials in Φ.

$$I = \sum_{i=0}^{m}[p_n(x_i) - y_i]^2 = \sum_{i=0}^{m}\left(\sum_{k=0}^{n}a_kx_i^k - y_i\right)^2 \quad (1)$$

If I is the minimum, $p_n(x)$ is a fitting polynomial. Obviously, $I = \sum_{i=0}^{m}\left(\sum_{k=0}^{n}a_kx^k - y_i\right)^2$ is an ill-conditioned equation with coefficients $a_0, a_1, \ldots, a_n$. To obtain the solutions of the coefficients, according to the essential condition of solving the ill-conditioned equation, we obtain the Eq. (2).

$$\frac{\partial I}{\partial a_j} = 2\sum_{i=0}^{m}\left(\sum_{k=0}^{n}a_kx_i^k - y_i\right)x_i^j = 0, \quad j = 0, 1, \ldots, n \quad (2)$$

The Eq. (2) can be rearranged into the following Eq. (3).

$$\sum_{k=0}^{n}\left(\sum_{i=0}^{m}x_i^{k+j}\right)a_k = \sum_{i=0}^{m}x_i^jy_i, \quad j = 0, 1, \ldots, n \quad (3)$$

The subsequent Eq. (4) is its matrix form.

$$\begin{bmatrix} m+1 & \sum_{i=0}^{m}x_i & \cdots & \sum_{i=0}^{m}x_i^n \\ \sum_{i=0}^{m}x_i & \sum_{i=0}^{m}x_i^2 & \cdots & \sum_{i=0}^{m}x_i^{n+1} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{i=0}^{m}x_i^n & \sum_{i=0}^{m}x_i^{n+1} & \cdots & \sum_{i=0}^{m}x_i^{2n} \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ \cdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{m}y_i \\ \sum_{i=0}^{m}x_iy_i \\ \cdots \\ \sum_{i=0}^{m}x_i^ny_i \end{bmatrix} \quad (4)$$

The coefficients $a_j(j = 0, \ldots, n)$ can be obtained by solving Eq. (4). Because m, n, x_i and $y_i(i = 0, \ldots, m)$ are given from the known condition. $p_n(x)$ can be formed in the following Eq. (4).

$$p_n(x) = \sum_{k=0}^{n}a_kx^k \quad (5)$$

2.2. *Biharmonic spline surface interpolation principle* [19–22]

The biharmonic spline surface interpolation was first proposed by Sandwell DT in 1987 [19], and then investigated by many other scholars [20–22]. The interpolation takes on the overall smoothness and good local performance [19]. The technique is not related to the distribution and number of control points. These properties are well suited for interpolating irregular and discrete data [19]. Biharmonic spline function passes through the discrete data points, generating a curve or surface. The generated curve or surface has minimum curvature because it meets biharmonic equation [19]. The following section is its relative principle.

For m dimensions of scattered distribution N control points $p_i(i = 1, \ldots, N)$, the problem is to find biharmonic function that passes through the given points. To find the biharmonic spline interpolation function, we can transform the problem into solving the Eqs. (6) and (7).

$$\nabla^4 w(p) = \sum_{j=1}^{N}a_j\delta(p - p_j) \quad (6)$$

$$w(p_i) = w_i \quad (7)$$

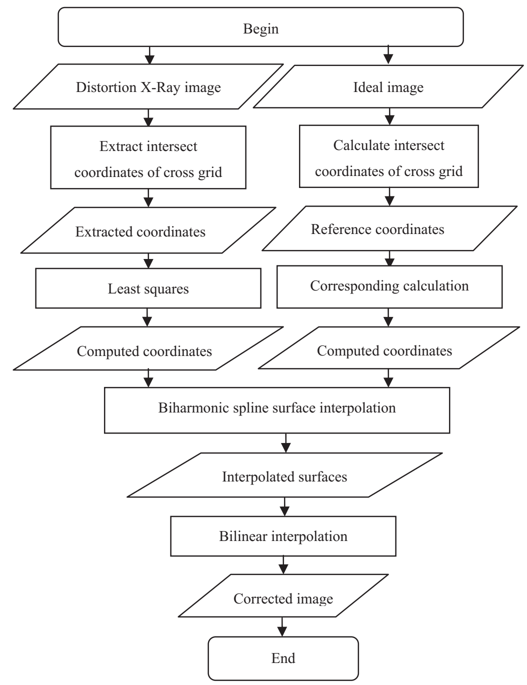

Fig. 1. The correction process.

Here ∇^4 is the biharmonic operator. p represents a position in m-dimensional space. $w(p)$ is the biharmonic function in the position p. $\varphi(p)$ is Delta function and it meets (6) and (7).

$$\delta(p) = \begin{cases} \infty & (p = 0) \\ 0 & (p \neq 0) \end{cases} \quad (8)$$

$$\int_{-\infty}^{+\infty}\delta(p)dp = 1 \quad (9)$$

The general solution meets Eqs. (6) and (7).

$$w(p) = \sum_{j=1}^{N}a_j\varphi_m(p - p_j) \quad (10)$$

Here $w(p)$ is the equation passing by the given points. Coefficients $a_j(j = 1, \ldots, N)$ meet the following linear Eq. (11).

$$w_i = \sum_{j=1}^{N}a_j\varphi_m(p_i - p_j) \quad (11)$$

3. The approach implement

With the process illustrated in Fig. 1, we can see that the proposed approach mainly includes the following several steps.

3.1. *Extract control points coordinates* [4,6]

In the selection, we use five processes to extract the coordinates of control points in the original image Fig. 2(a) and its intensities exist between 0 and 255. To obtain the image background Fig. 2(b), the original image intensities are replaced with opposite intensities (subtract its intensities from 255). If the intensities differences between the original image and dealt image are greater than the fixed value (150), we assign 255 to the position in the original image and others remain unchanged. After the removed background image, we use Otsu algorithm to segment into image