Optik 126 (2015) 2518–2522

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Optical soliton solutions for the variable coefficient modified Kawahara equation

Ahmet Bekir [a,*], Özkan Güner [b], Halis Bilgil [c]

[a] Eskisehir Osmangazi University, Art-Science Faculty, Department of Mathematics-Computer, Eskisehir, Turkey
[b] Cankiri Karatekin University, Faculty of Economics and Administrative Sciences, Department of International Trade, Cankiri, Turkey
[c] Aksaray University, Art-Science Faculty, Department of Mathematics, Aksaray, Turkey

ARTICLE INFO

Article history:
Received 30 April 2014
Accepted 9 June 2015

Keywords:
Optical solitons
Exact solution
The variable-coefficient modified Kawahara equation

ABSTRACT

In this paper, we obtain the 1-soliton solutions of the variable-coefficient modified Kawahara equation (VCMKE). The dark optical as well as bright optical soliton solutions were found related to the model considered in this study. The solitary wave ansatz method is used to carry out the integration.

1. Introduction

By now, more and more nonlinear wave equations not only in mathematical but also in various fields have been used. These wave equations appear in a great array of contexts such as, fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations [1]. Optical solitons is one of the important areas of research in the field of Nonlinear Optics. This area of research has made an enormous progress especially in the past decades.

In recent years, new exact solutions may help to find new phenomena. A variety of powerfull methods, such as the tanh–sech method [2,3], extended tanh method [4,5], sine–cosine method [6,7], homogeneous balance method [8,9], first integral method [10,11], $\left(\frac{G'}{G}\right)$-expansion method [12,13], trial method [14,15] and F-expansion method [16,17] were used to solve nonlinear dispersive and dissipative problems. These approaches possess powerful features that make the determination of multiple soliton solutions practical for a wide class of nonlinear evolution equations. Moreover, the solitary ansatz method has been used for the determination of the dark and bright soliton solutions.

The soliton-like solutions for nonlinear PDEs is too important for extensive applications in many physics areas. Envelope solitons are stable nonlinear wave packets that preserve their shape when propagating in a nonlinear dispersive medium. Two different types of envelope solitons, dark optical (topological) and bright optical (non-topological) soliton solution, can propagate in nonlinear dispersive media. Compared with the bright soliton which is a pulse on a zero-intensity background, the dark soliton appears as an intensity dip in an infinitely extended constant background [18].

Interest in variable-coefficient nonlinear evolution equations has grown steadily in recent years. This is due to the fact that most of the real nonlinear wave equations possess variable coefficients. Further, nonlinear physical equations with variable coefficients are more realistic in various physical situations than their constant-coefficient counterparts. The reason for this is that constant-coefficient models can only describe the propagation of wave groups in perfect systems.

The variable-coefficient modified Kawahara equation (VCMKE)

$$u_t + a(t)u^2u_x + b(t)u_{xxx} + c(t)u_{xxxxx} = 0, \tag{1}$$

where $a(t)$, $b(t)$ and $c(t)$ are arbitrary funtions of t.

Thanks to the efforts of many researchers, certain types of nonlinear modified Kawahara equation have been investigated and solved [19,20], which arise in modeling of various physical phenomena, are studied by Lie group analysis and generalized (G'/G)-expansion method in [21].

* Corresponding author. Tel.: +90 222 2393750; fax: +90 222 2393578.
E-mail addresses: abekir@ogu.edu.tr (A. Bekir), ozkanguner@karatekin.edu.tr (Ö. Güner), halis@aksaray.edu.tr (H. Bilgil).

http://dx.doi.org/10.1016/j.ijleo.2015.06.051
0030-4026/

2. Applications of solitary wave ansatz method

The solitary wave ansatz method proposed by Biswas [22] and Triki et al. [23] is particularly notable in its power and applicability in solving nonlinear problems, and it has been successfully applied to many kinds of nonlinear partial differential equations [24–31].

2.1. The bright optical soliton solution of the variable coefficient modified Kawahara equation (VCMKE)

Bright optical solitons are also known as bell-shaped solitons and non-topological solitons. We start the analysis by assuming a solitary wave ansatz of the form [32,33]

$$u(x,t)=\lambda \operatorname{sech}^{p}(\eta(x-vt)), \tag{2}$$

where the parameters $\lambda=\lambda(t)$ is the amplitude of the soliton, $\eta=\eta(t)$ is the inverse width of the soliton and $v=v(t)$ is the velocity of the soliton. The unknown p will be determined during the course of derivation of the solutions of equation (1). From the ansatz (2), it is possible to find

$$u_t=\frac{d\lambda}{dt}\operatorname{sech}^{p}\tau-\lambda p\left\{x\frac{d\eta}{dt}-\frac{d(t\eta v)}{dt}\right\}\operatorname{sech}^{p}\tau\tanh\tau, \tag{3}$$

$$u^2u_x=-\lambda^3 p\eta\operatorname{sech}^{3p}\tau\tanh\tau \tag{4}$$

$$u_{xxx}=-\lambda(p\eta)^3\operatorname{sech}^{p}\tau\tanh\tau+p(p+1)(p+2)\eta^3\lambda\operatorname{sech}^{p+2}\tau\tanh\tau, \tag{5}$$

$$\begin{aligned}u_{xxxxx}=&-p^5\lambda\eta^5\operatorname{sech}^{p}\tau\tanh\tau+2p(p+1)(p+2)\left(p^2+2p+2\right)\\&\times\eta^5\lambda\operatorname{sech}^{p+2}\tau\tanh\tau-p(p+1)(p+2)(p+3)(p+4)\\&\times\eta^5\lambda\operatorname{sech}^{p+4}\tau\tanh\tau,\end{aligned} \tag{6}$$

where $\tau=\eta(x-vt)$.

Substituting Eqs. (3)–(6) into Eq. (1), we have

$$\begin{aligned}&\frac{d\lambda}{dt}\operatorname{sech}^{p}\tau-\lambda p\left\{x\frac{d\eta}{dt}-\frac{d(t\eta v)}{dt}\right\}\operatorname{sech}^{p}\tau\tanh\tau\\&-a(t)\lambda^3p\eta\operatorname{sech}^{3p}\tau\tanh\tau\\&-b(t)\lambda(p\eta)^3\operatorname{sech}^{p}\tau\tanh\tau+b(t)p(p+1)(p+2)\eta^3\lambda\operatorname{sech}^{p+2}\tau\tanh\tau\\&-c(t)p^5\lambda\eta^5\operatorname{sech}^{p}\tau\tanh\tau+2c(t)p(p+1)(p+2)\left(p^2+2p+2\right)\eta^5\lambda\operatorname{sech}^{p+2}\tau\tanh\tau\\&-c(t)p(p+1)(p+2)(p+3)(p+4)\eta^5\lambda\operatorname{sech}^{p+4}\tau\tanh\tau\\&=0,\end{aligned} \tag{7}$$

Now, from (7), matching the exponents of $\operatorname{sech}^{3p}\tau\tanh\tau$ and $\operatorname{sech}^{p+4}\tau$, one gets

$$3p=p+4, \tag{8}$$

so that

$$p=2. \tag{9}$$

Setting the coefficients of $\operatorname{sech}^{p}\tau$ terms in Eq. (7) to zero, we have

$$\frac{d\lambda}{dt}=0, \tag{10}$$

so that the amplitude λ is constant.

$$\lambda(t)=\lambda_0. \tag{11}$$

From (7), setting the coefficients of $\operatorname{sech}^{2p+2}\tau$ and $\operatorname{sech}^{p+4}\tau$ terms to zero we obtain

$$b(t)p(p+1)(p+2)\eta^3\lambda+2c(t)p(p+1)(p+2)(p^2+2p+2)\eta^5\lambda=0, \tag{12}$$

that leads to:

$$\eta(t)=\sqrt{\frac{-b(t)}{2(p^2+2p+2)c(t)}}, \tag{13}$$

Substituting Eq. (9) into (13), we obtain

$$\eta(t)=\sqrt{\frac{-b(t)}{20c(t)}}. \tag{14}$$

Remarkably, the inverse width of the bright solitary wave solution in Eq. (14) exists that $b(t)c(t)<0$.

The t dependence of the soliton velocity $v(t)$ is found from setting the coefficients of $\operatorname{sech}^{p}\tau\tanh\tau$ terms in Eq. (7) to zero, we have

$$-\lambda p\left\{x\frac{d\eta}{dt}-\frac{d(t\eta v)}{dt}\right\}-b(t)\lambda(p\eta)^3-c(t)p^5\lambda\eta^5=0, \tag{15}$$

which can be rewritten as follows:

$$-\lambda p\left\{x\frac{d\eta}{dt}-\frac{d(t\eta v)}{dt}+b(t)p^2\eta^3+c(t)p^4\eta^5\right\}=0. \tag{16}$$

Taking into account the fact that the soliton parameter $v(t)$ we want to determine from Eq. (16) is a function of time, one can split Eq. (16) into two equations as follows:

$$\frac{d\eta}{dt}=0, \tag{17}$$

$$-\frac{d(t\eta v)}{dt}+b(t)p^2\eta^3+c(t)p^4\eta^5=0, \tag{18}$$

which gives some calculations

$$\eta(t)=\eta_0 \tag{19}$$

$$v(t)=\frac{1}{\eta(t)t}\int_0^t\left\{p^2b(t\prime)\eta^3(t\prime)+p^4c(t\prime)\eta^5(t\prime)\right\}dt\prime \tag{20}$$

where η_0 is an integral constant related to the initial pulse inverse width. Substituting Eq. (9) into (20), we obtain

$$v(t)=\frac{1}{\eta(t)t}\int_0^t\left\{4b(t\prime)\eta^3(t\prime)+16c(t\prime)\eta^5(t\prime)\right\}dt\prime \tag{21}$$

Having obtained the expressions for the pulse parameters λ, η and v, we construct a family of the one-soliton-type, exact analytic solutions for Eq. (1) as follows:

$$u(x,t)=\lambda\operatorname{sech}^{p}(\eta(x-vt)). \tag{22}$$

Substituting Eqs. (9), (19) and (21) into Eq. (22) we have

$$u(x,t)=\lambda_0\operatorname{sech}^2\left\{\eta_0\left(x-\left(\frac{1}{\eta(t)t}\int_0^t\left\{4b(t\prime)\eta^3(t\prime)+16c(t\prime)\eta^5(t\prime)\right\}dt\prime\right)t\right)\right\}. \tag{23}$$

2.2. The dark soliton solution of the variable coefficient modified Kawahara equation (VCMKE)

In this section, we are interested in finding the dark soliton solution (expressed as hyperbolic tangent function), as defined in [34], for the considered VCMKE equation (1). Dark solitons are also known as topological optical solitons in the context of Nonlinear Optics.