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Allan variance method for gyro noise analysis using weighted least square algorithm

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ABSTRACT

The Allan variance method is an effective way of analyzing gyro's stochastic noises. In the traditional implementation, the ordinary least square algorithm is utilized to estimate the coefficients of gyro noises. However, the different accuracy of Allan variance values violates the prerequisite of the ordinary least square algorithm. In this study, a weighted least square algorithm is proposed to address this issue. The new algorithm normalizes the accuracy of the Allan variance values by weighting them according to their relative quantitative relationship. As a result, the problem associated with the traditional implementation can be solved. In order to demonstrate the effectiveness of the proposed algorithm, gyro simulations are carried out based on the various stochastic characteristics of SRS2000, VG951 and CRG20, which are three different-grade gyros. Different least square algorithms (traditional and this proposed method) are applied to estimate the coefficients of gyro noises. The estimation results demonstrate that the proposed algorithm outperforms the traditional algorithm, in terms of the accuracy and stability.

1. Introduction

The navigation system provides essential information for the vehicle's control system. Inertial navigation system (INS) is one of the most popular navigation systems, which can measure vehicle's position, velocity and attitude without receiving signals from external equipments [1]. Gyros are the critical components of the INS. They can provide angular velocity information for the INS [2].

The navigation accuracy of the INS is greatly affected by gyros' errors. The analysis of gyros' errors has two purposes. One is to evaluate gyros' performances and list them on their specifications. This work is usually done by producers. Another is to provide the reference for the filter parameters setting of the INS-based integrated navigation system. This work is usually done by users. Gyro error falls into two categories: the deterministic errors and the stochastic errors [3,4]. The deterministic errors include bias and scale factor errors, whose models and the parameters are easy to obtain. The stochastic errors contain a variety of complex stochastic noises, which are difficult to model. Several methods including the Allan variance method can be utilized to analyze the stochastic noises [5,6].

The Allan variance method, which is the effective way to analyze stochastic noises, is widely applied in the field of gyro signal analysis, including laser gyros [7], fiber optical gyros (FOGs) [8], and micro electro mechanical systems (MEMS) gyros [9]. The Allan variance method mainly consists of two steps [10]: the first step is to calculate the Allan variance values from the gyro's sample data; the second step is to estimate the gyro's stochastic coefficients by fitting Allan variance values. Both steps affect the coefficients estimation accuracy, and this work focuses on the second step.

At present, the ordinary least square algorithm is usually applied to fit Allan variance values [11], but problem remains. Allan variance values are essentially random variables with different variance. Therefore, the accuracy of each Allan variance value is different from others [12]. The different accuracy violates the prerequisite of the ordinary least square algorithm. As a result, it is not valid to estimate the gyro's stochastic coefficients using the ordinary least square algorithm. Some research work [13,14] tried to remove the Allan variance values which had large estimation errors and used the rest part of the values in the ordinary least square algorithm. However, the solution partly decreased the errors but not completely solved the problem, because the estimation errors of the rest Allan variance estimates were still not in the same level.

In this study, a weighted least square algorithm is proposed to solve the problem. In the new algorithm, the Allan variance values are weighted according to their relative estimation accuracy, so that

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the Allan variance values are normalized and the fitting errors are reduced.

The rest of this paper is organized as follows. In Section 2 we briefly introduce the commonly used gyro's stochastic models and their Allan variances, and then the fitting process of the model coefficients is analyzed using the ordinary least square. In Section 3 the defects of the traditional algorithm are discussed and the new algorithm is described in detail. Simulations are carried out in Section 4, which are based on the noise characteristics of three types of gyros, i.e. SRS2000, VG951 and CRG20. The performances of different least square algorithms are compared.

2. Allan variance method and gyro stochastic noises

There are usually several types of stochastic noises in the gyro's signal. Since the Allan variance method can identify different types of noises in the given data set, it is appropriate for the gyro noise study.

2.1. Gyros' stochastic noises

The gyro's stochastic error is complex, which is usually described as the combination of several stochastic noises. Quantization noise, angle random walk, bias instability, rate random walk and rate ramp are five common stochastic noises existing in the gyro's output [15]. The sources of the five noises are the electronic components in gyros, which lead to stochastic errors and decrease the gyros' accuracy. The five noises' characteristics, which can be described by their power spectrums, are different from each other. In this paper, the coefficients of the five noises are identified by using the Allan variance method.

2.2. Allan variance values

The first step of the Allan variance method is to calculate Allan variance values from the gyro's data. The gyro's sample data series is denoted as ω_k, the sampling frequency is f and the sample size is N. The calculation process of Allan variance values is as follows:

(1) m ($m = 1, 2, \ldots, [N/2]$) data points are made as one cluster, so we can get $J = [N/m]$ clusters, where $[x]$ represents the floor of x.
(2) To take the average of each cluster as $\bar{\omega}_k(m)$:

$$\bar{\omega}_k(m) = \frac{1}{m}\sum_{i=1}^{m}\omega_{(k-1)\times m+i}, \quad k = 1, 2, \ldots, \left[\frac{N}{2}\right]. \tag{1}$$

(3) $\tau_m = m/f$ is defined as the correlation time of each cluster and the Allan variance is defined as

$$\sigma^2(\tau_m) = \frac{1}{2(J-1)}\sum_{k=1}^{J-1}\left[\bar{\omega}_{(k+1)}(m) - \bar{\omega}_k(m)\right]^2. \tag{2}$$

If the gyro signal is composed of the five noises, i.e. quantization noise, angle random walk, bias instability, rate random walk and rate ramp, whose coefficients are denoted as Q, N, B, K and R, its Allan variance can be derived as [16]

$$\sigma^2(\tau_m) = \frac{3Q^2}{\tau_m^2} + \frac{N^2}{\tau_m} + (0.664B)^2 + \frac{K^2\tau_m}{3} + \frac{R^2\tau_m^2}{2}. \tag{3}$$

It can be seen from Eq. (3) that the Allan variance is a function of the correlation time, which can be described by a log–log curve called the Allan variance curve. The properties of different stochastic noises can be reflected in the curve.

2.3. Model parameters identification

The purpose of the Allan variance method is to identify the noises' coefficients. In Eq. (3), σ^2 and τ_m are known variables and Q, N, B, K, R are five unknown coefficients. Intuitively, we may use five Allan variance values, plugging into Eq. (3) to solve these five unknown coefficients algebraically. However, each Allan variance value has the uncertainty (see Section 2.2), which may have negative impact on the solution of the coefficient. To overcome the uncertainty, curve-fitting or regression methods can be utilized here to estimate the coefficient instead of solving it algebraically. Since the Allan variance is a polynomial function of the five unknown coefficients, a polynomial fitting method (the ordinary least square algorithm) can be used to obtain the coefficients [13]. The fitting model can be written as

$$\begin{cases} \sigma_1^2 = 3Q^2\tau_{m1}^{-2} + N^2\tau_{m1}^{-1} + (0.664B)^2 + K^2\tau_{m1}/3 + R^2\tau j_{m1}^2/2 + w_1 \\ {\sigma_2}^2 = 3Q^2\tau_{m2}^{-2} + N^2\tau_{m2}^{-1} + (0.664B)^2 + K^2\tau_{m2}/3 + R^2\tau_{m2}^2/2 + w_2 \\ \vdots \\ {\sigma_i}^2 = 3Q^2\tau_{mi}^{-2} + N^2\tau_{mi}^{-1} + (0.664B)^2 + K^2\tau_{mi}/3 + R^2\tau_{mi}^2/2 + w_i \end{cases}. \tag{4}$$

where σ_i is the calculated Allan variance value, w_i is the corresponding estimation error which is assumed as white noise, and τ_{mi} is the correlation time. Define the system in the matrix form, we can have

$$\boldsymbol{y} = \boldsymbol{X}\beta + \boldsymbol{e}. \tag{5}$$

where

$$\boldsymbol{y} = \begin{bmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \vdots \\ \sigma_i^2 \end{bmatrix}, \quad \boldsymbol{X} = \begin{bmatrix} \tau_{m1}^{-2} & \tau_{m1}^{-1} & 1 & \tau_{m1} & \tau_{m1}^2 \\ \tau_{m2}^{-2} & \tau_{m2}^{-1} & 1 & \tau_{m2} & \tau_{m2}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \tau_{mi}^{-2} & \tau_{mi}^{-1} & 1 & \tau_{mi} & \tau_{mi}^2 \end{bmatrix},$$

$$\beta = \begin{bmatrix} 3Q^2 \\ N^2 \\ (0.664B)^2 \\ K^2/3 \\ K^2/2 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{e} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_i \end{bmatrix}. \tag{6}$$

According to the ordinary least square algorithm, the coefficients matrix β can be solved as

$$\beta = \left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}\boldsymbol{X}^T\boldsymbol{y}. \tag{7}$$

where $\boldsymbol{X}^T$ is the transposed matrix of $\boldsymbol{X}$.

The gyro's performance can be evaluated by its noises' coefficients. In the real-time INS, gyro's main stochastic noises are usually extended to the filter equation as state variables, and then the stochastic noises can be estimated and compensated. The navigation accuracy will be improved accordingly [17].

3. Limitations of the ordinary least square algorithm and its solution

From Eq. (2), the Allan variance values are calculated based on finite sample data. The Allan variance accuracy varies with the number of samples (clusters). The different Allan variance accuracy violates the important prerequisite of the same level of accuracy in the ordinary least square algorithm. In this section, the traditional solution is introduced and its defects are analyzed, and then an improved solution is proposed.